Solutions to Selected Problems from 8.3

23. $\int \sqrt{5+4x-x^2}\,dx$

$$= \int \sqrt{-(x^2-4x+4)+5+4}\,dx$$

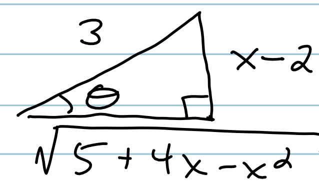
$$= \int \sqrt{9-(x-2)^2}\,dx$$

$a = 3$

$u = x-2 = 3\sin\theta$

$dx = 3\cos\theta\,d\theta$

$$= \int \sqrt{9-9\sin^2\theta}\;3\cos\theta\,d\theta$$



$$= 3\int \sqrt{1-\sin^2\theta}\cdot 3\cos\theta\,d\theta$$

$$= 9\int \cos^2\theta\,d\theta$$

$$= 9\cdot\frac{1}{2}\int 1+\cos 2\theta\,d\theta$$

$$= \frac{9}{2}\left[\theta + \frac{1}{2}\sin 2\theta\right] + C$$

$$= \frac{9}{2}\left[\theta + \frac{1}{2}\cdot 2\sin\theta\cos\theta\right] + C$$

$$= \boxed{\frac{9}{2}\left[\arcsin\left(\frac{x-2}{3}\right) + \frac{x-2}{3}\cdot\frac{\sqrt{5+4x-x^2}}{3}\right] + C}$$

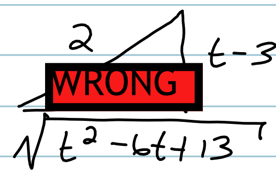
24. $\displaystyle\int \frac{1}{\sqrt{t^2-6t+13}}\,dt$

$$= \int \frac{1}{\sqrt{t^2-6t+9+13-9}}\,dt$$

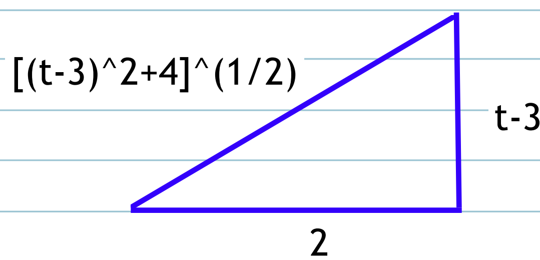
$$= \int \frac{1}{\sqrt{(t-3)^2+4}}\,dt$$

$a = 2$
$u = t-3 = 2\tan\theta$
$d\theta = 2\sec^2\theta\, d\theta$

$$= \int \frac{1}{\sqrt{4\tan^2\theta+4}}\, 2\sec^2\theta\, d\theta$$



$$= \int \frac{1}{\cancel{2}\sqrt{\tan^2\theta+1}}\,\cancel{2}\sec^2\theta\, d\theta$$



$$= \int \frac{1}{\sqrt{\sec^2\theta}}\sec^2\theta\, d\theta$$

$$= \int \sec\theta\, d\theta$$

$$= \ln\left|\sec\theta + \tan\theta\right| + C$$

$$= \boxed{\ln\left|\frac{2}{\sqrt{t^2-6t+13}} + \frac{t-3}{\sqrt{t^2-6t+13}}\right| + C}$$

WRONG

= ln|[(t-3)^2+4]^(1/2)/2+(t-3)/2]|+C

29. $$\int x\sqrt{1-x^4}\,dx$$

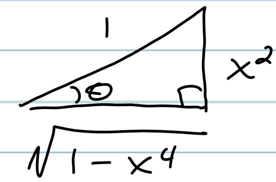
$a = 1$

$u = x^2 = \sin\theta$

$2x\,dx = \cos\theta\,d\theta$

$$dx = \frac{\cos\theta\,d\theta}{2x}$$

$$= \int \cancel{x}\sqrt{1-\sin^2\theta}\;\frac{\cos\theta\,d\theta}{2\cancel{x}}$$



$$= \frac{1}{2}\int \sqrt{\cos^2\theta}\;\cos\theta\,d\theta$$

$$= \frac{1}{2}\int \cos^2\theta\,d\theta$$

$$= \frac{1}{2}\cdot\frac{1}{2}\int 1+\cos 2\theta\,d\theta$$

$$= \frac{1}{4}\left[\theta + \frac{1}{2}\sin 2\theta\right] + C$$

$$= \frac{1}{4}\left[\theta + \frac{1}{\cancel{2}}\cdot\cancel{2}\sin\theta\cos\theta\right] + C$$

$$= \boxed{\frac{1}{4}\left[\arcsin(x^2) + x^2\cdot\sqrt{1-x^4}\right] + C}$$