Within ε of independence: An attempt to produce independent proof-writers via an IBL approach in a real analysis course

Getting Students Involved in Writing Proofs
JMM 2011, New Orleans, LA

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What is Inquiry-Based Learning (IBL)?

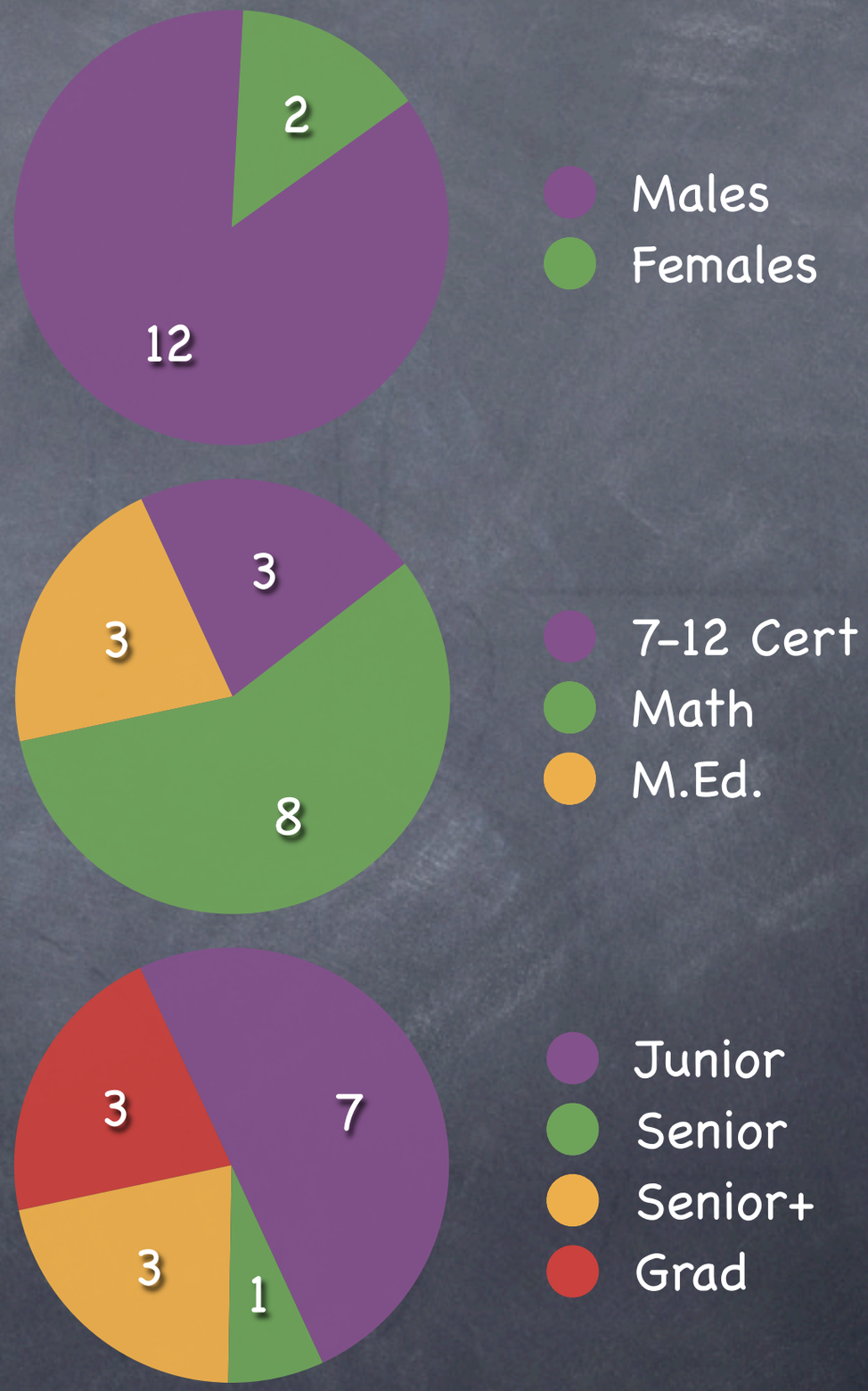
- Also known as Modified Moore Method, after R.L. Moore
- According to the Academy of Inquiry-Based Learning:
  - IBL is a teaching method that engages students in sense-making activities.
  - Students are given tasks requiring them to solve problems, conjecture, experiment, explore, create, & communicate.
  - Rather than showing facts or a clear, smooth path to a solution, the instructor guides students via well-crafted problems through an adventure in mathematical discovery.
- Often involves very little lecturing
- Students are responsible for guiding acquisition of knowledge

Motivation for study

- For 3 consecutive semesters, I taught an intro to proof course @ PSU
- 1st two iterations taught via traditional lecture-based approach, where students only engaged in the process of proof while working on homework/exams
- 3rd instance taught using IBL with heavy emphasis on collaboration (used wiki, students collaborated on exams)
- When I taught an abstract algebra course containing students from both styles, anecdotal evidence suggested that students taught via IBL were stronger proof-writers & more independent as learners
- Inspired by apparent effectiveness of IBL, chose to adopt this approach & study it with Angie

Demographics of students

- PSU is a regional comprehensive university located in NH
- 14 students
- Not required of majors
- However, we offer very few upper-division math courses
- All but 2 students had taken at least 1 proof-based course
- 8 had prior IBL experience
- All students knew what format of course would be



Structure of Course

- 3 times per week, 50 minutes each
- Used Ted Mahavier's *Analysis* notes (available for **FREE** from Journal of Inquiry-Based Learning in Mathematics)
- Axioms of Reals → Fundamental Theorem of Calculus
- Notes contain 81 theorems, all but 5 assigned, students proved all but 4 of assigned
- 2-3 theorems assigned each class
- Nearly all class time devoted to students presenting proposed proofs to assigned theorems
- Encouraged to collaborate, even on take-home exams!
- Students turned in written work for all theorems (re-writes allowed)
- Allowed to "fix" proofs during presentations

Presentations

- 1 proof at a time, typically presenters volunteered
- Presenter required to:
  - use proper grammar & write in complete sentences
  - explain reasoning & address questions from audience
- Allowed to start over or pass to someone else if “stuck”
- Audience expected to ask questions of presenter & seek clarification, allowed to give hints/advice

My Role as Instructor

- Keep class on task, facilitate discussion, provide feedback & mini-lectures as needed
- Mini-lectures: less than 20 min/week
- Provide constant positive feedback!
- Ask leading questions of presenter & audience when “stuck”
- Interject when students on “wrong track”

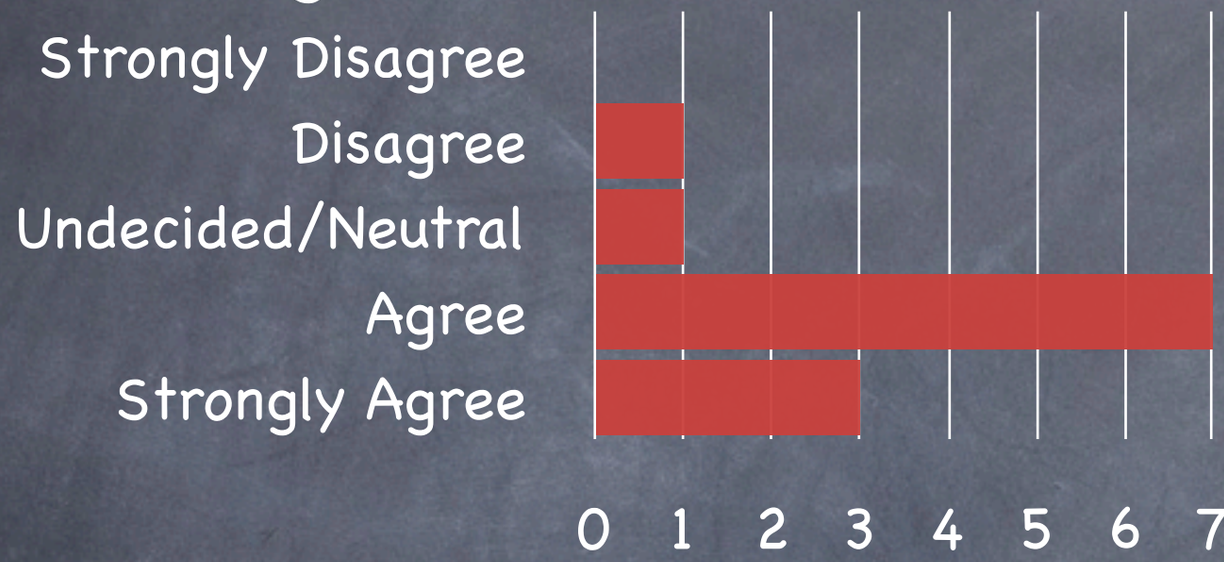
Overview of Study

- 52 five-point Likert scale & 21 open-ended questions
- Optional, given during last week of classes, implemented via Google Docs form
- 87% response rate (12 of 14 students)
- Questions fell into 9 categories:
  - Teaching method
  - Collaboration
  - Perception of ability
  - Perspectives of proofs
  - Attitude
  - Approach to proofs
  - Motivation
  - Comfort level
  - Demographics

- ✓ 18 questions of 73
- ✓ In general, responses were positive
- ✓ Here is a snapshot

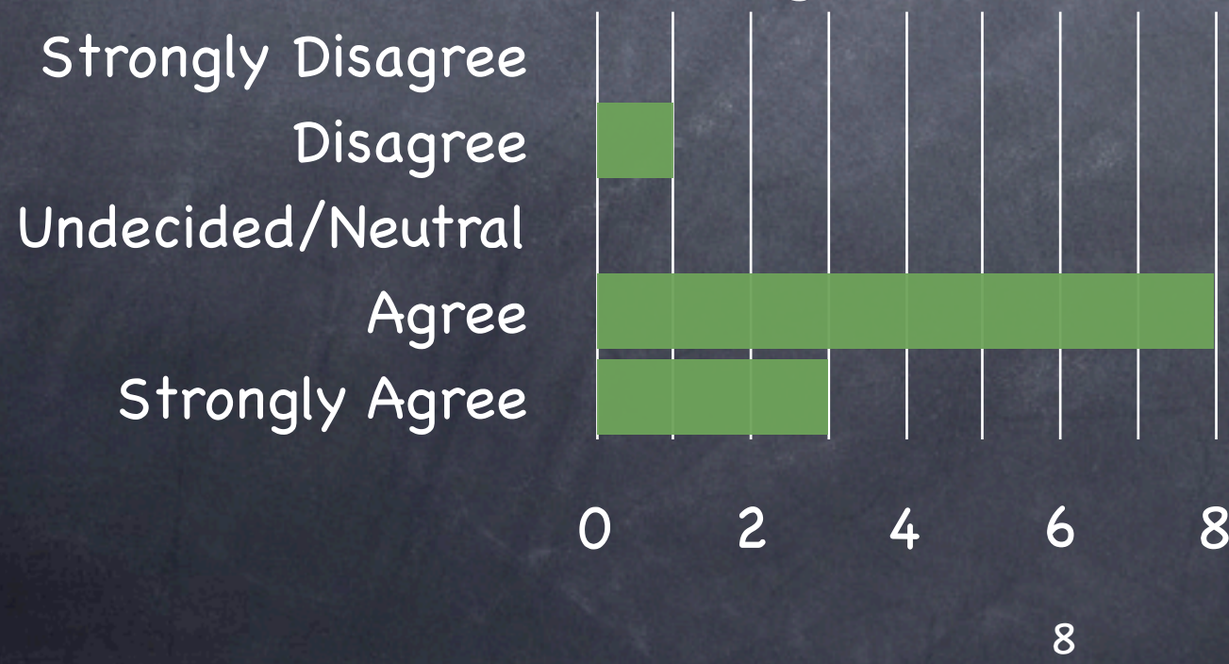
Response to Teaching Method

- The way this class is taught has been beneficial to my learning.



83% Agree/Strongly Agree

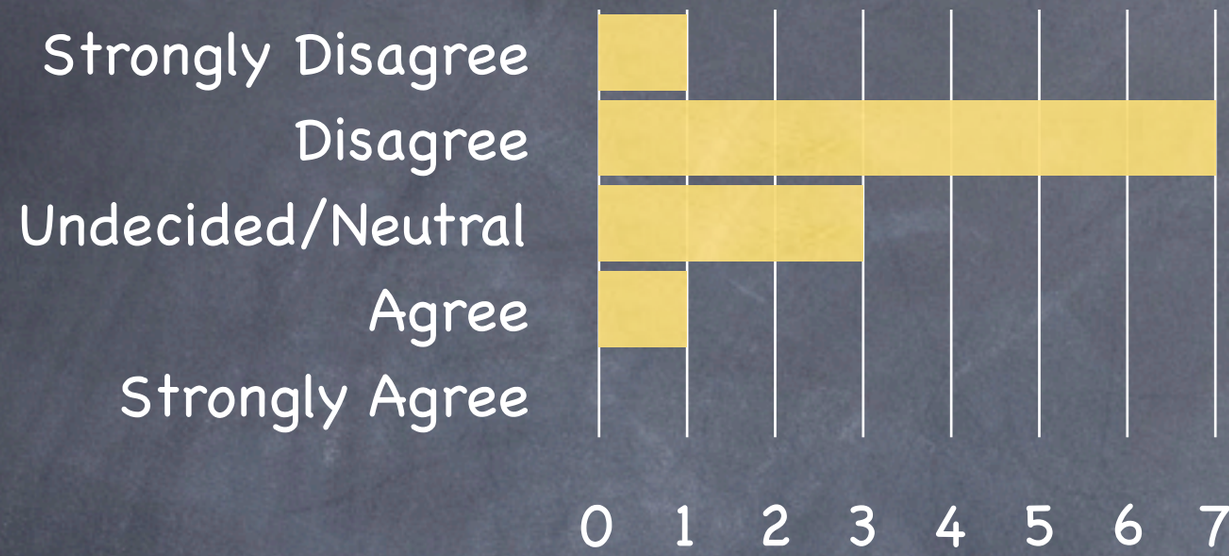
- Watching other students present their proposed proofs is useful to my learning.



92% Agree/Strongly Agree

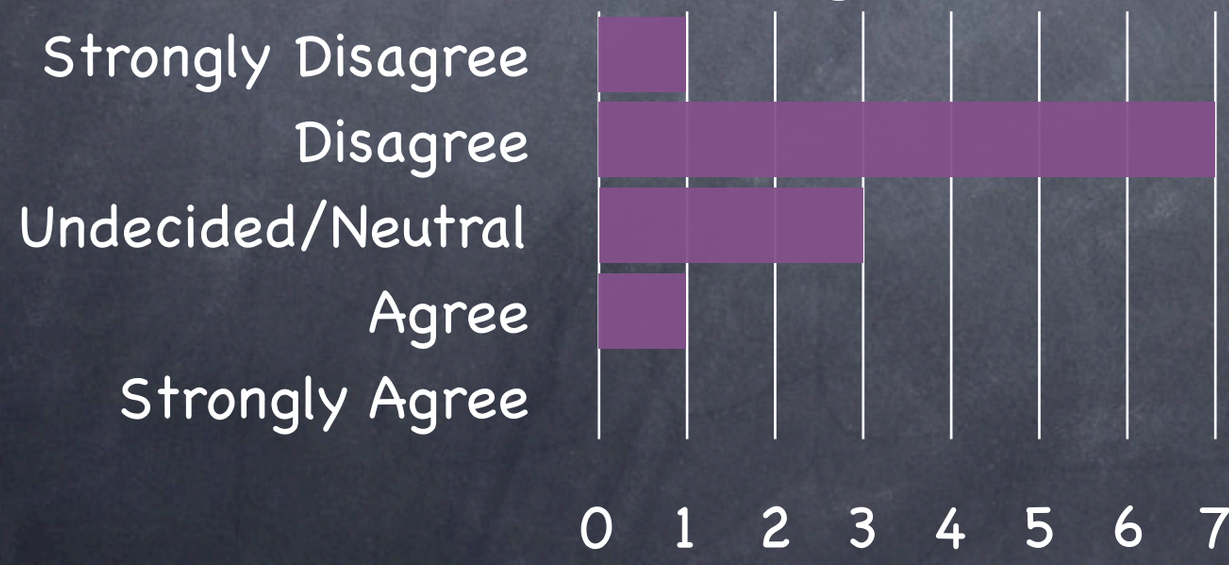
Response to Teaching Method (continued)

- The teacher's role in a mathematics class is to lecture.



67% Disagree/Strongly Disagree
25% Undecided/Neutral

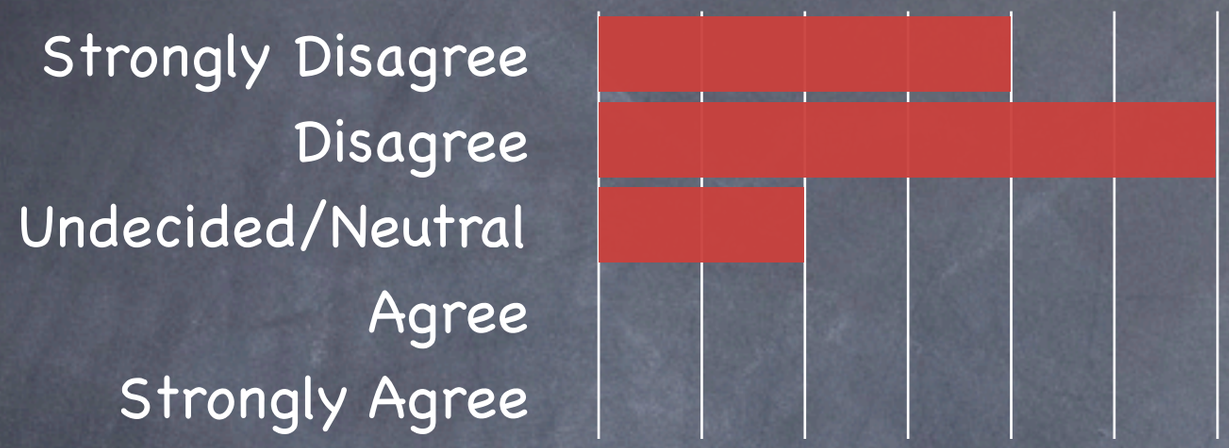
- When other students present incorrect proofs on the board, it is confusing.



67% Disagree/Strongly Disagree
25% Undecided/Neutral

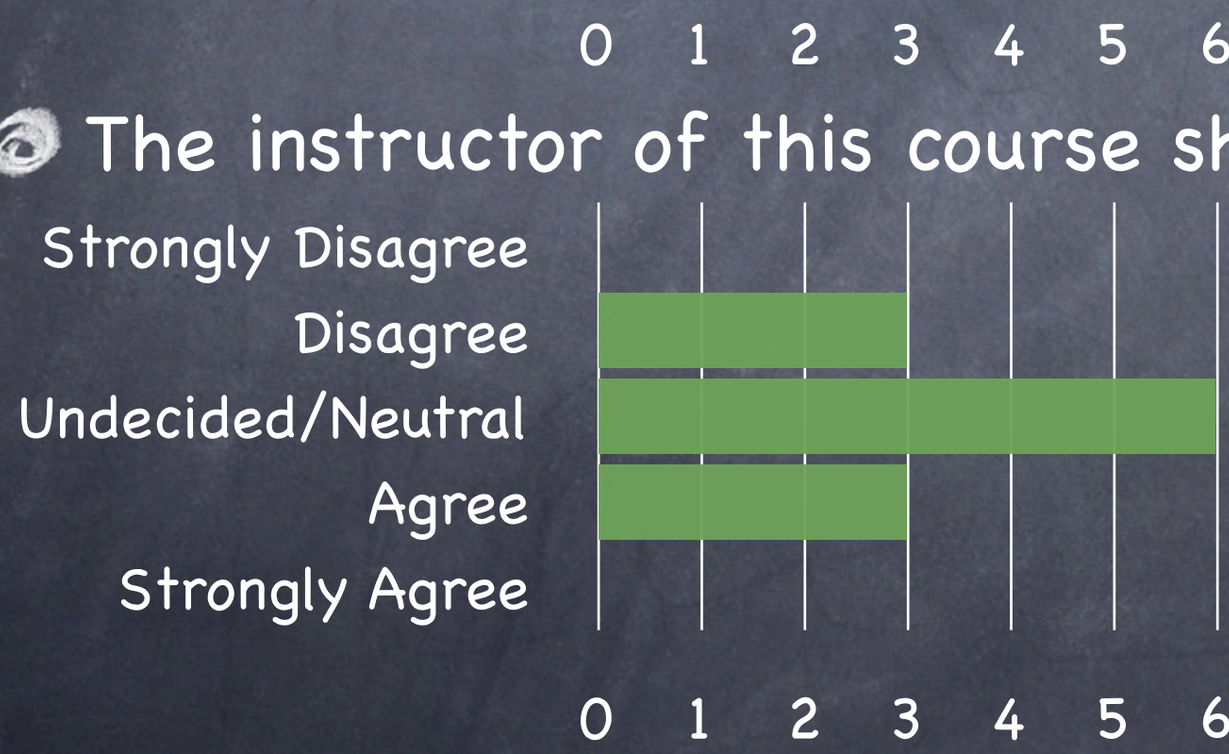
Response to Teaching Method (continued)

- I see no benefit to me presenting my proposed proof at the board.



83% Disagree/Strongly Disagree

- The instructor of this course should lecture more.



50% Undecided/Neutral & the rest is split evenly between Disagree & Agree

Response to Teaching Method (continued)

- How is your real analysis course taught?
  - "Very hands on. Forces you to gain working knowledge on the subject matter."
  - "By students attempting the proof, and volunteering to show their proofs at the board with the teacher intervening when the student struggles just long enough."
- How does this compare to the way your other mathematics courses have been taught?
  - "Most math courses are less open and involve a great deal of leacture. They assign more homework but I spent a much larger amount of time on my Analysis HW due to the need to understand over the need to learn the presented algorithm..."
  - "More collaborative, interactive and relaxed."

Response to Teaching Method (continued)

- Please describe any advantages to the method with which this class was taught that you haven't mentioned already.
  - "This course deals with a subject matter that is best explored rather then presented...I particularly liked the move into the conference room where we all began to feel more like participants in a think-tank instead of students waiting for knowledge to be handed over."
  - "The advantage of participating in a class like this is I am more confident of myself being an independent learner."
  - "It forces students to take on concepts on their own, and be independent."

Response to Teaching Method (continued)

- Please describe any disadvantages to the method with which this class was taught that you haven't mentioned already.
  - “i don't like doing proofs”
  - “Students who may have fallen behind may feel even more uncomfortable with writing proofs.”
  - “More designed for the self-motivated, self-learner which I am not too good at being.”

Future Work

- Angie & I are working on a paper that addresses other categories:
  - Teaching method
  - Collaboration
  - Perception of ability
  - Perspectives of proofs
  - Attitude
  - Approach to proofs
  - Motivation
  - Comfort level
  - Demographics

Pilot Study: Present quantitative data supporting the effectiveness of a collaborative IBL approach, & present qualitative data describing student perception of knowledge acquisition with regards to proof.

- Conduct larger study over multiple semesters with pre- & post-tests.

Potential Improvements

- Streamline notes by eliminating extraneous problems
- More "give an example of..." & "prove or disprove"
- Visual structure of connections between theorems
- More summarizing of proof
- More structured in-class activities
- Increase use of GeoGebra applets, have students create applets
- Lurch

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