Collaborative peer review between two IBL number theory courses

Scholarship of Teaching and Learning
in Collegiate Mathematics
JMM 2012, Boston, MA

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What is inquiry-based learning (IBL)?

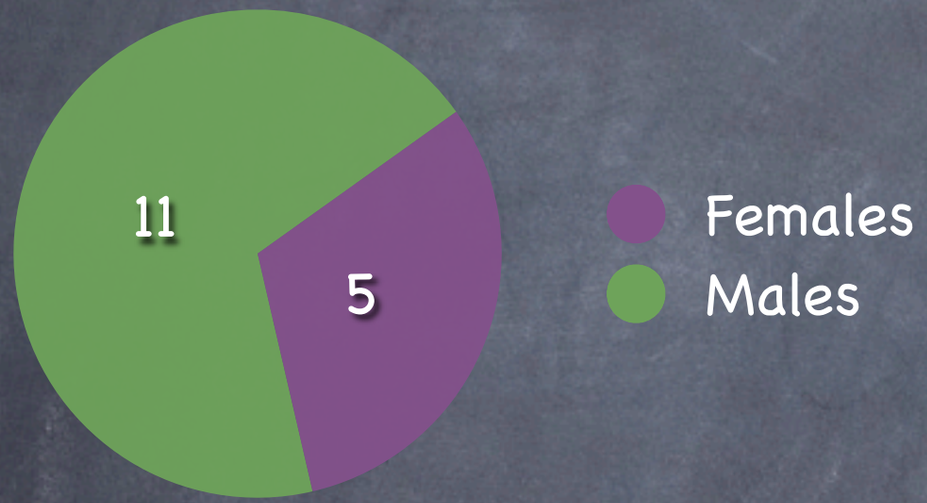
- According to the Academy of Inquiry-Based Learning:
  - IBL is a teaching method that engages students in sense-making activities.
  - Students are given tasks requiring them to solve problems, conjecture, experiment, explore, create, & communicate.
  - Rather than showing facts or a clear, smooth path to a solution, the instructor guides students via well-crafted problems through an adventure in mathematical discovery.
- Often involves very little lecturing

Overview of project

- Andy Schultz (Wellesley College), Angie Hodge (University of Nebraska at Omaha), & I attended 2010 IBL Workshop
- During workshop, we kicked around several ideas about how to increase collaboration in IBL courses
- Andy & I discovered we were both teaching Number Theory in Spring 2011
- Chose to adopt IBL approach & use same book (Number Theory Through Inquiry by Marshall, Odell, & Starbird)
- Developed plan to incorporate anonymous peer review between 2 classes
- Asked Angie to help develop survey to study student perception of peer review project

Plymouth State University

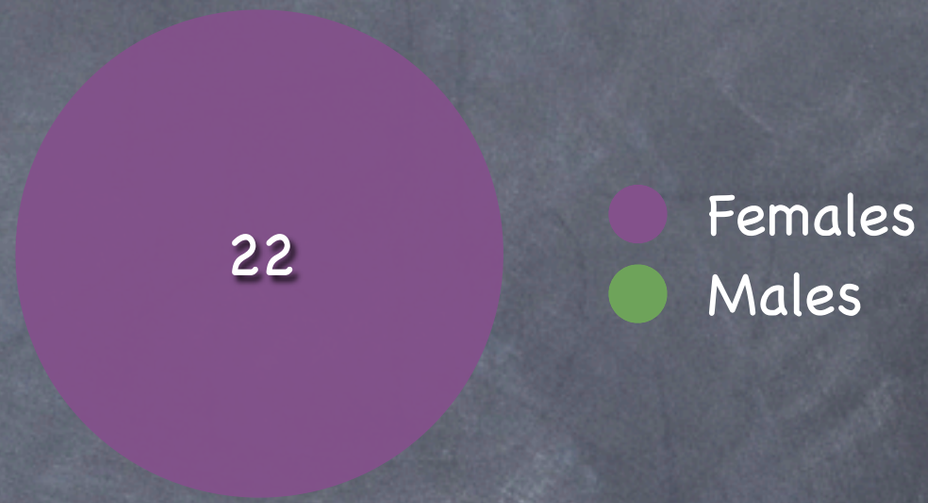
- PSU is a regional comprehensive university located in NH
- 16 students



- 12 had taken at least 1 proof-based course
- 2 chemistry majors
- 6 had prior IBL experience
- met 3 hours per week

Wellesley

- Wellesley is a selective liberal arts college for women
- 22 students



- 14 had taken at least 1 proof-based course
- 6 non-math majors
- 0 had prior IBL experience
- met 4 hours per week & started 1 week earlier

Nuts & bolts of each course

- Nearly all class time devoted to students presenting proposed solutions/proofs to assigned exercises
- Students required to write in complete sentences & use proper grammar, discussion included comments about style & grammar
- Students encouraged to collaborate
- Daily Homework: assigned each class meeting, graded on ✔-system, students allowed to annotate work (with felt tip pen) in light of presentation & discussion
- Weekly Write-ups: students submitted 2 formally written proofs each week, typed (typically LaTeX), usually subset of previous week's Daily Homework

Description of peer review project

- Anonymous peer review between classes occurred twice
- Students in both classes were given same instructions
- Proofs chosen to submit for review were questions from take-home exams
- Students were required to type their proofs & referee reports (submit as PDF)
- Students were provided template for writing reports
- For both exchanges:
  - Each PSU student sent 2 proofs to Wellesley
  - Each Wellesley student sent 1 proof to PSU
  - Each student reviewed 2 proofs

Description of peer review project (continued)

- Each referee report consisted of 3 parts:
  - Summary: general comments about overall correctness & clarity of exposition
  - Detailed Report: specific comments by line number
  - Numerical Evaluation: score for each of 2 categories:
    - Correctness: validity of argument
    - Style: grammar, punctuation, & overall presentation

Grade	Criteria
4	work is perfect
3	work is nearly perfect, but there are some minor errors
2	work has at least one significant problem
1	work contains many significant errors and/or doesn't seem to address question

- Score given by referee had no impact on other student
- Referee reports were graded by instructor

My impression of peer review project

- Overall, we were pleased
- Students did an excellent job of refereeing
- Worked better than in-house peer reviews would
- Some unexpected items:
  - Managing all of the files was difficult!
  - What's your reaction for most of the referee reports you get? We didn't anticipate the intensity of student reactions.
  - Students were super picky about notation & style!
- Act of refereeing more useful than receiving feedback

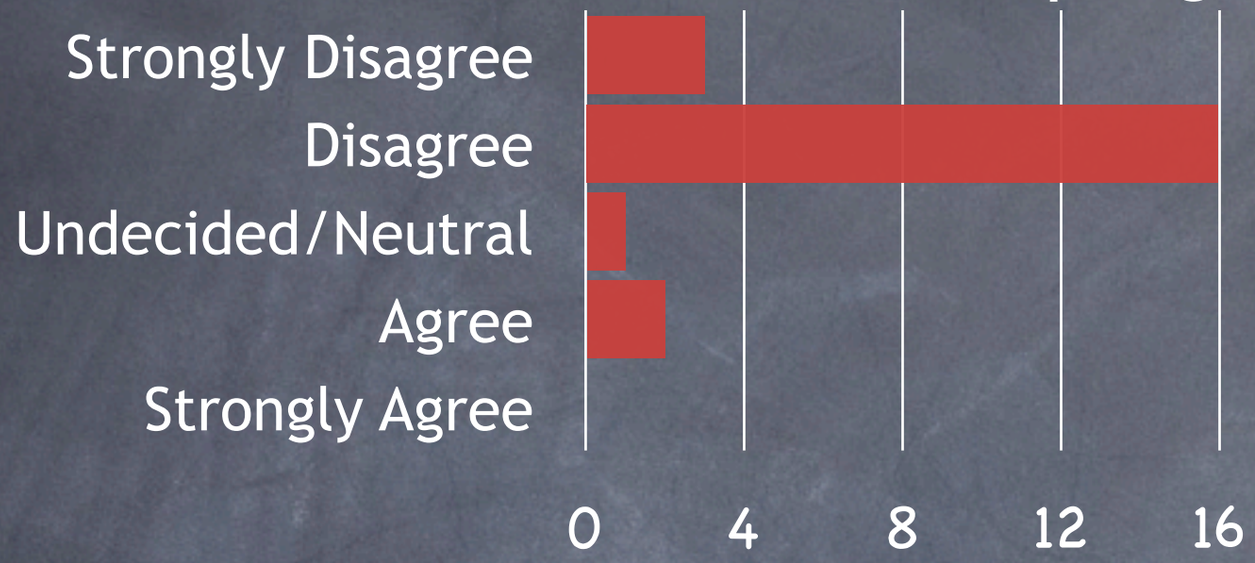
Overview of study

- Optional pre- & post-test survey was given to students in both classes
- Implemented via Google Docs form
- Questions fell into 10 categories (one of which was peer review)
- Post-test contained 17 questions (14 Likert scale, 3 open-ended) addressing peer review
- Response rate for post-test:
  - PSU: 14/16
  - Wellesley: 8/22
  - Overall: 22/38

- ✓ Low response rate
- ✓ But responses were insightful
- ✓ Here is a snapshot

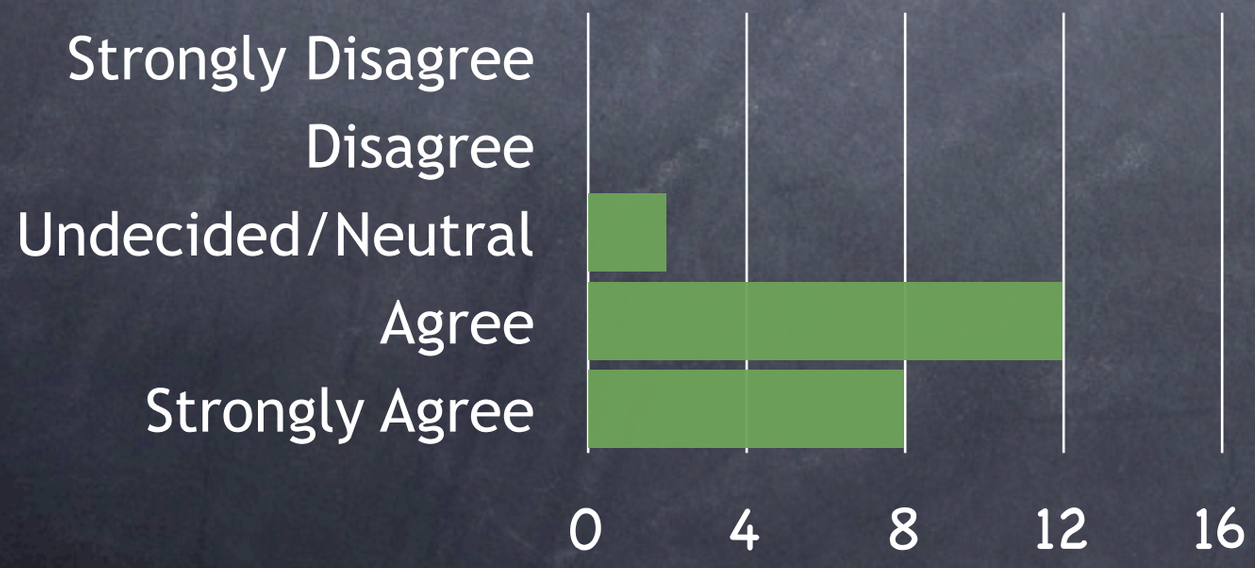
Student response to peer review

- I see no benefit to me critiquing another student's work.



86% Disagree/Strongly Disagree

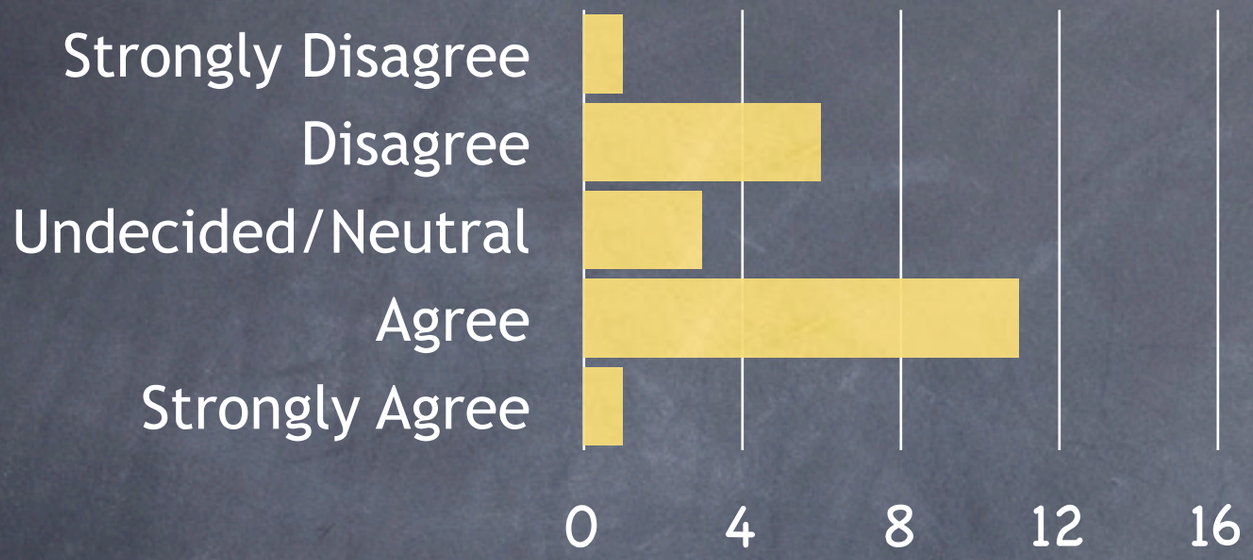
- I am more capable of evaluating the validity of another student's written work & providing appropriate feedback than I was when this course began.



91% Agree/Strongly Agree

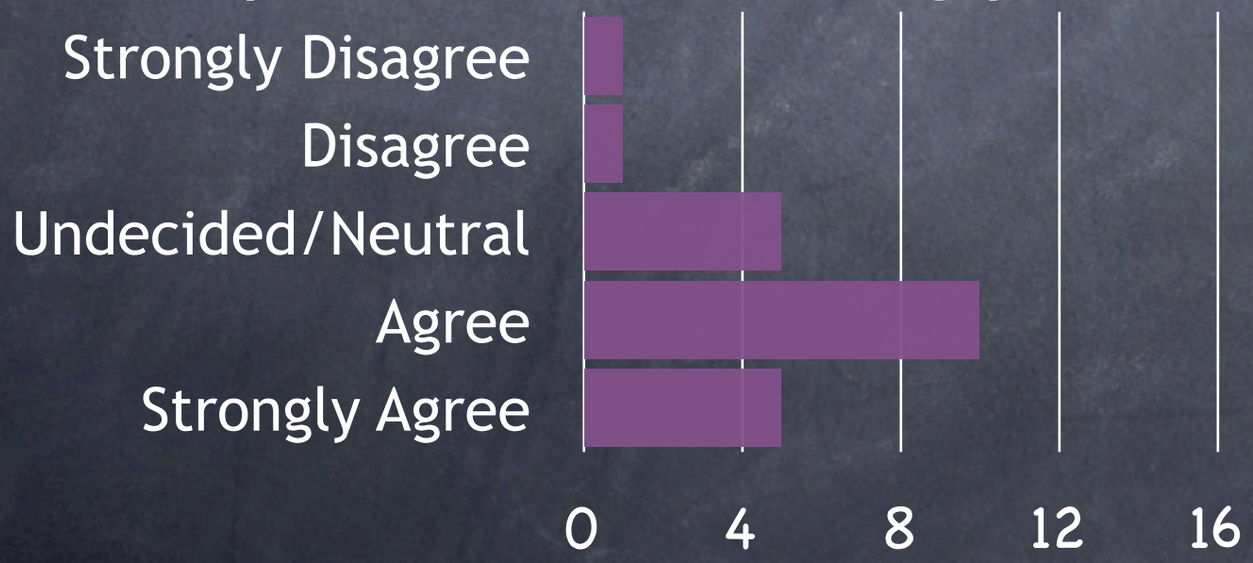
Student response to peer review (continued)

- The feedback I received from written peer reviews was helpful.



Thoughts on this seem divided

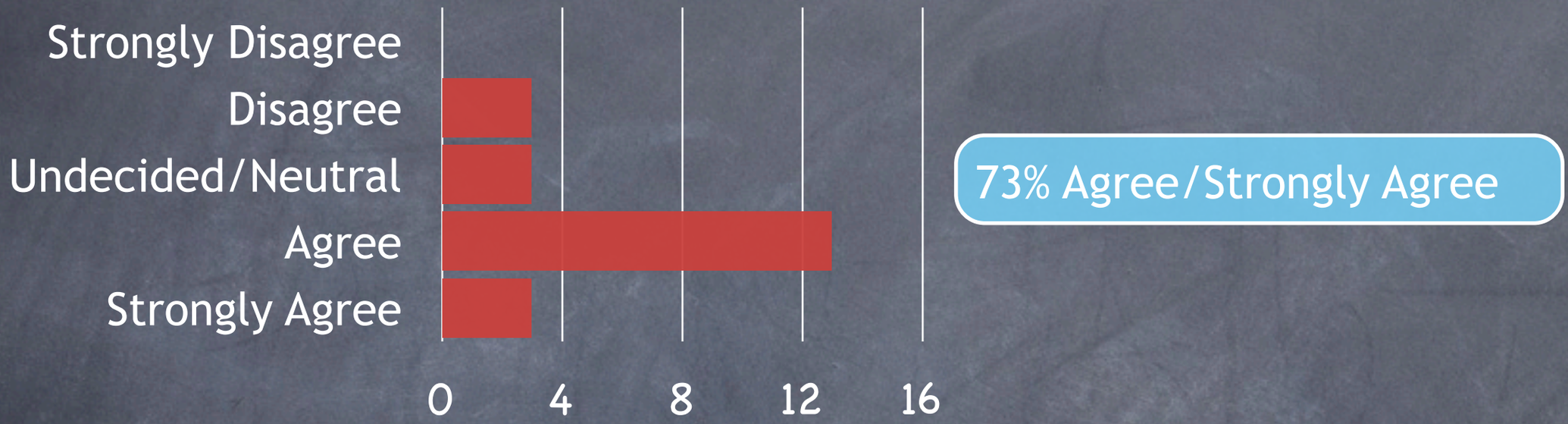
- The process of writing peer reviews was more helpful than the process of receiving peer reviews.



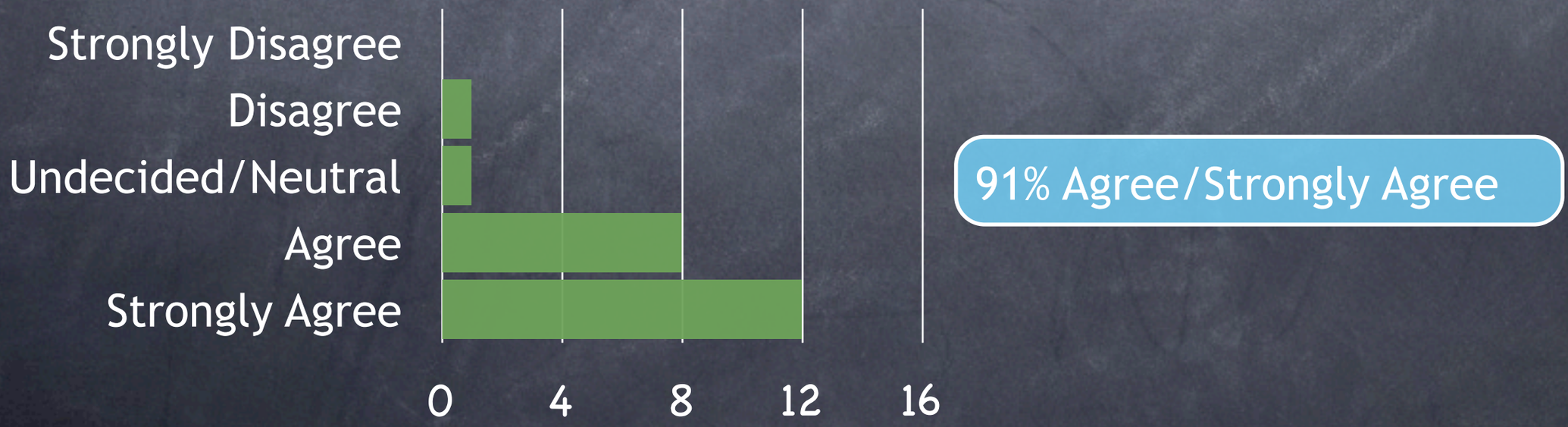
68% Agree/Strongly Agree
23% Undecided/Neutral

Student response to peer review (continued)

- After spending time critiquing other students' work, I am more capable of evaluating my own proofs critically.



- As a result of this course, I have improved as a proof-writer.



Student response to peer review (continued)

Describe your role in the peer review process for this class. What did it mean for you to "peer review?" How did you use the feedback provided by others? How was providing feedback for others helpful to you?

"The peer review process was good because we got to see how other people write proofs. Also, if I ever write a paper, I'd know what to expect when it got reviewed."

"It meant that I got to critique another student's proof who may have learned in a different style. It was interesting to see what student's at the other school did versus what students in my class might have done. I used the feedback I received to improve my proofs, and hopefully the feedback I provided did the same."

Student response to peer review (continued)

Which aspect of the peer review process did you find to be the most beneficial? Writing formal reviews or receiving feedback? Please explain.

"writing formal reviews. helps me pinpoint errors and faulty logic, also helps me pick out things i like."

"Writing the review was more beneficial. You had to really look at a proof and figure out why something was right or wrong, and give an explanation. Not just 'this doesn't look right.'"

"I found receiving them to be much more beneficial. I am not confident enough in my proof validating abilities to review other's proofs, but can accept constructive criticism on my own proofs with ease by a stranger..."

Ideas for improvement

- Remove numerical score?
- Do more than 2?
- Increase difficulty of problems chosen for submission
- Perhaps let students choose which problems to submit
- Prepare students for negative feedback
- Provide them with more examples of referee reports (including negative ones!)
- Find better way to manage exchange of proofs & referee reports (Annotum?)

Thank you! Please contact us if you have questions:

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Example

Theorem 2 If p and q are distinct primes and a is a natural number such that $(a, pq) = 1$ then $a^{(p-1)(q-1)} \equiv 1 \pmod{pq}$.

Proof. Assume that p and q are distinct primes and a is a natural number such that $(a, pq) = 1$ Then by theorem 4.32 $a^{\phi(pq)} \equiv 1 \pmod{pq}$. Note that $\phi(pq)$ is equal to the number of natural numbers less than or equal to pq that are relatively prime to pq. There are pq numbers in pq but not all of them are relatively prime to pq. By multiplication we know that there are p multiples of q in pq and q multiples of p in pq. Since $(p, pq) \neq 1$ and $(q, pq) \neq 1$, you have to subtract the number of multiples of p and the number of multiples of q from pq to get ϕ. So we have $\phi(pq) = pq - p - q$. But, you have to add one to the expression because the number pq is in the q multiples of p as well as in the p multiples of q so you can't count it twice. Therefore, $\phi(pq) = pq - p - q + 1$. We see that

$$\begin{aligned} a^{\phi(pq)} &\equiv 1 (\text{mod } pq) \\ a^{pq-p-q+1} &\equiv 1 (\text{mod } pq) \\ a^{(p-1)(q-1)} &\equiv 1 (\text{mod } pq). \end{aligned}$$

Therefore $a^{(p-1)(q-1)} \equiv 1 \pmod{pq}$. □

Example (continued)

Referee's Report for Theorem 07B

May 3, 2011

1 Summary Report

Score (Style) – 3

This proof occasionally had an informal tone that was inappropriate. I understood the entire logical process, but the language should be more mathematical and less casual.

Score (Mathematical Correctness) – 3

The mathematical concepts behind this proof were all correct, but some portions needed a more formal explanation. Instead of explaining everything in layman's terms, the writer should cite specific mathematical concepts.

2 Detailed Report

Style

- (Line 1) I would change the wording of this sentence slightly by either inserting "that" before " a is a natural number…" or changing "assume that" to "let".
- (Line 3) When you say "there are pq numbers in pq", it is not clear. Say something like "we know there are k integers such that $1 \leq k \leq pq$ instead.
- (Line 4) I would not cite "by multiplication". Try to use something about divisibility instead, or better yet, use Euler's totient function since both p and q are prime.
- (Lines 5,6) Don't use "you" – it makes your proof seem very informal. Try to explain this in a less informal, colloquial way.
- (Line 11) I might mention that we are factoring the power of $a^{pq-p-q+1}$ in this step.