Diagram calculus for the Temperley–Lieb algebra

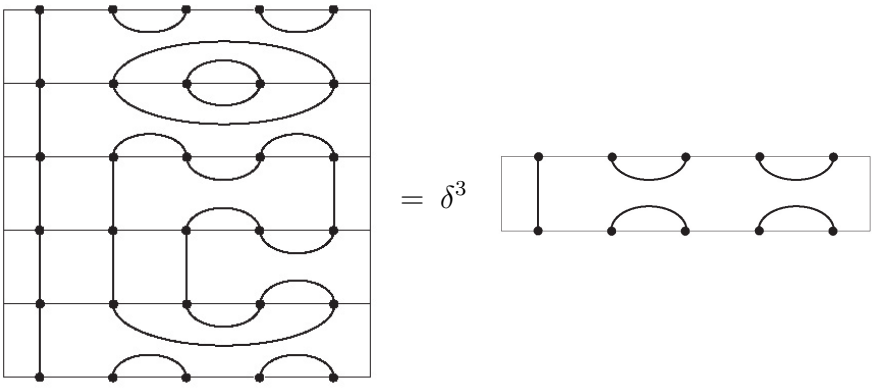
Dana Ernst, Plymouth State University

Northeastern Section Meeting of the MAA
Bentley University, November 22, 2008

Abstract: I have spent quite a bit of time working on mathematics while sipping coffee at various coffee houses in Boulder, CO and now Plymouth, NH. It is fairly common for people to look over my shoulder and wonder what exactly I'm working on:

> **Stranger:** "Excuse me sir, but what the heck are you drawing?"
>
> **Me:** "Oh, these? I'm doing mathematics."
>
> **Stranger:** "No, really, what are you doing?"
>
> **Me:** "This really is mathematics. I'm trying to use these pictures to cleverly model an abstract mathematical object that is otherwise difficult to work with."

One aspect of my research involves trying to prove that certain associative algebras can be faithfully represented using "diagrams." These diagrammatic representations are not only nice to look at, but they also help us recognize things about the original algebra that we may not otherwise have noticed. In this talk, we will discuss one example of a diagram algebra. In particular, we will introduce the diagram calculus for the Temperley–Lieb algebra, denoted TL_n. This algebra, invented by Temperley and Lieb in 1971, is a certain finite dimensional associative algebra, depending on a parameter δ, which arose in the context of statistical mechanics in physics. We will show that TL_n has dimension equal to the nth Catalan number and also discuss its relationship to the symmetric group S_n.



An example of multiplication of diagrams in TL_n.