1 Introduction to Mathematics

1.1 A Taste of Number Theory

In this section, we will work with the set of integers, $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$. The purpose of this section is to get started with proving some theorems about numbers and study the properties of $\mathbb{Z}$.

Definition 1.1. An integer n is **even** if $n = 2k$ for some integer k.

Definition 1.2. An integer n is **odd** if $n = 2k + 1$ for some integer k.

Theorem 1.3. The sum of two consecutive integers is odd.

Theorem 1.4. If n is even, then n^2 is even.

Problem 1.5. Prove or Disprove: the sum of an even number and an odd number is odd.

Problem 1.6. Prove or Disprove: the product of an odd number and an even number is odd.

Problem 1.7. Prove or Disprove: the product of an odd number and an odd number is odd.

Problem 1.8. Prove or Disprove: the product of two even numbers is even.

Definition 1.9. An integer n **divides** the integer m, written $n|m$, if and only if there exists an integer k such that $m = nk$. In the same context, we may also write that m **is divisible by** n. (Note: In this section on number theory, we allow addition, subtraction, and multiplication. Division is not allowed since an integer divided by an integer may result in a number that is not an integer. The upshot: don't write $\frac{m}{n}$). When you feel the urge to divide, switch to an equivalent formulation using multiplication.

Theorem 1.10. Suppose n and a are integers. If n divides a, then n divides a^2.

Theorem 1.11. The sum of any three consecutive integers is always divisible by three.

Theorem 1.12. Assume a, m, and n are integers. Suppose a divides m and a divides n. Then a divides $m + n$.

Problem 1.13. Assume a, b, and m are integers. Determine whether the following statement holds sometimes, always, or never. If the number ab divides m, then a divides m and b divides m.

Problem 1.14. Prove or Disprove: If a and b are integers and a divides b^2, then a divides b.

Theorem 1.15. If a, b, and c are integers where a divides b and b divides c, then a divides c.

This work is an adaptation of notes written by Stan Yoshinobu of Cal Poly and Matthew Jones of California State University, Dominguez Hills.