Weekly Homework 1: Review of Calculus I and II

Names:

Goal

The goal of this assignment is to review some of the main topics from Calculus I and II and to get your brain thinking about calculus again. Don't panic if there are a couple questions that you don't remember how to do. However, if you find yourself struggling significantly, then we should talk.

Directions

In groups of 2–4 (I do not want anyone working alone), answer each of the following questions in the space provided. You only need to turn in one lab per group (make sure you put everyone's name on this sheet). Feel free to consult your notes and textbook. This assignment is due by 5PM on **Thursday, February 2** and is worth 10 points.

Exercises

For you those of you that had me last semester, you should recognize most of the following problems from your final exam.

1. Consider the following function.

$$f(x) = \begin{cases} \cos x, & x > 0 \\ e^{x+1}, & x \leq 0 \end{cases}$$

   For (a)–(d), evaluate the given expression. If an expression does not exist, specify whether it equals ∞, $-\infty$, or simply does not exist (in which case, write DNE). You do *not* need to justify your answers.

   (a) $\lim\limits_{x\to 0^-} f(x) =$ ____________

   (b) $\lim\limits_{x\to 0^+} f(x) =$ ____________

   (c) $\lim\limits_{x\to 0} f(x) =$ ____________

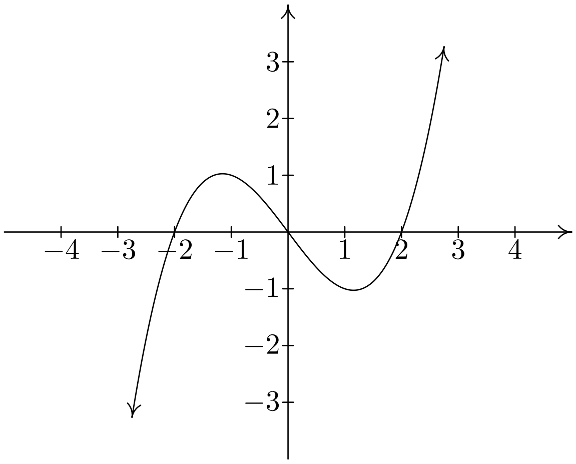
   (d) $f(0) =$ ____________

2. Evaluate each of the following limits. If a limit does not exist, write DNE. Sufficient work must be shown and proper notation should be used. In particular, you should write limits where appropriate and if you make use of L'Hospital's Rule, you should make it explicit where you are doing so. Give *exact answers*.

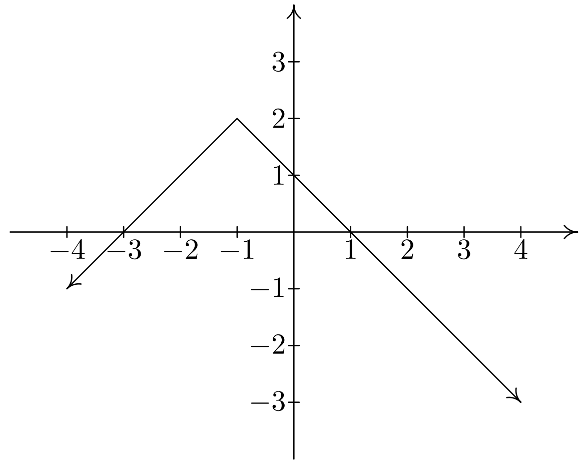
   (a) $\lim\limits_{x\to -2} \dfrac{x^2+2x}{x^2-4}$

   (b) $\lim\limits_{x\to 0^+} x \ln x$

3. Consider the following graphs for functions f and g. Assume that the graph of f is symmetric about the origin. Using the graphs, evaluate each of the following expressions. If an expression does not exist, write DNE. You do *not* need to justify your answer.



Graph of f



Graph of g

(a) $g(1) =$ _____________.

(b) $g'(1) =$ _____________.

(c) $g'(-1) =$ _____________.

(d) $f'(0)$ is (i) positive, (ii) negative, or (iii) 0. (Circle the correct answer.)

(e) Suppose $H(x) = f(g(x))$. Then $H'(1)$ is (i) positive, (ii) negative, or (iii) 0. (Circle the correct answer.)

(f) $\int_{-2}^{2} f(x)\, dx$ is (i) positive, (ii) negative, or (iii) 0. (Circle the correct answer.)

(g) $\int_{-3}^{3} g(x)\, dx$ is (i) positive, (ii) negative, or (iii) 0. (Circle the correct answer.)

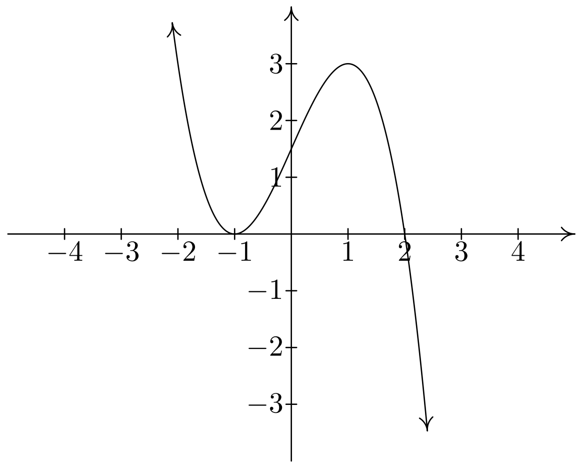
4. Differentiate each of the following functions. You do *not* need to simplify your answers, but sufficient work must be shown to receive full credit. If you make a mistake in an intermediate step while simplifying, it will count against you.

   (a) $y = \dfrac{3x^2 - x + 4}{1 - x^2}$

   (b) $A(x) = \displaystyle\int_0^{x^2} \sqrt{1 + \ln t}\, dt$

5. Find an *equation* of the tangent line to the graph of $f(x) = \sin x$ at $x = \pi/4$. It does not matter what form your equation takes, but you should use exact values for coefficients and constants.

6. Let f be a differentiable function. Suppose that the following graph is the graph of the *derivative* of f (i.e., the graph of f'). You do *not* need to justify your answers.



Graph of f'

(a) Find the x-coordinates of all points on the graph of f where the tangent line is horizontal.

(b) Find the (open) intervals, if any, on which f is increasing.

(c) Find the (open) intervals, if any, on which f is decreasing.

7. Use appropriate calculus techniques to find the absolute maximum and absolute minimum values of the function $f(x) = xe^{-x}$ on the interval $[0, 2]$. Sufficient work must be shown.

8. The shock-waves from an earthquake on the ocean floor radiate out in the form of a circle on the surface of the ocean from its epicenter. If the radius of the shock-waves is increasing at a rate of 3 miles per second, what is the rate of change of the area enclosed by the radiating shock-waves when the radius is 2 miles? Give an exact answer. Your answer should be labeled with appropriate units.

9. Evaluate each of the following integrals. Sufficient work must be shown. In the case of a definite integral, you should give an *exact* answer.

   (a) $\displaystyle\int \frac{x}{x^2+1}\,dx$

   (b) $\displaystyle\int_1^e \frac{x^2+1}{x}\,dx$

(c) $\displaystyle\int_0^1 \frac{1}{x^2+1}\, dx$

(d) $\displaystyle\int \frac{e^x}{\sqrt{1-e^{2x}}}\, dx$

(e) $\displaystyle\int \frac{1}{x^2\sqrt{x^2-9}}\, dx$

(f) $\int \sin^3 x \cos^2 x \; dx$

(g) $\int x \cos x \; dx$

(h) $\int \frac{x^3}{x^2+5} \; dx$

(i) $\displaystyle\int \frac{x+5}{x^2+x-2}\,dx$

(j) $\displaystyle\int_2^{\infty} \frac{1}{4+x^2}\,dx$ (If the integral converges, give an *exact* answer.)

10. Let $A(x) = \displaystyle\int_0^x \sin^2 t\ dt$. Determine where A attains a maximum value on the interval $[0, \pi]$. Justify your answer. Arguing using an appropriate picture is sufficient, but not mandatory.

11. Setup (but do *not* evaluate) an integral that would determine the volume of the solid obtained by revolving the region bounded by the given graphs about the indicated line.

    (a) $y = 2 - x^2$ and $y = x^2$, about the x-axis.

    (b) $f(x) = 1$ and $g(x) = x^2$, about the line $x = 2$.

12. Find the area of one loop of the graph of $r = 3\sin(5\theta)$.

13. Consider the parametric curve given by

$$x = \frac{8}{3}t^{3/2}, \qquad y = 2t - t^2$$

Find the arc length for $1 \leq t \leq 2$.