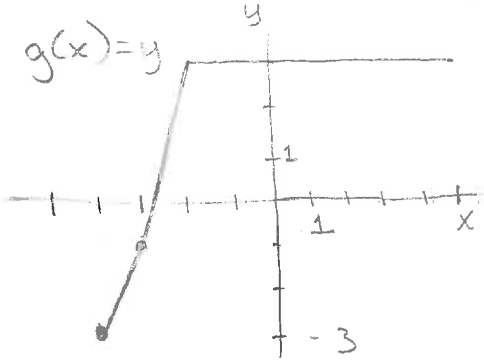
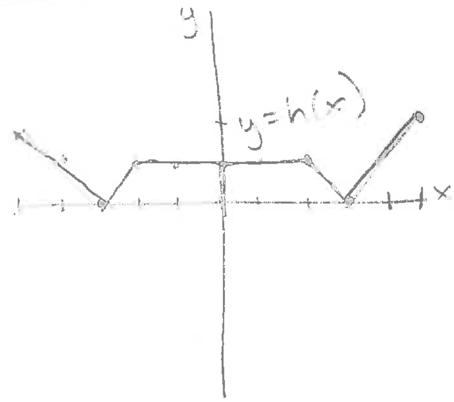
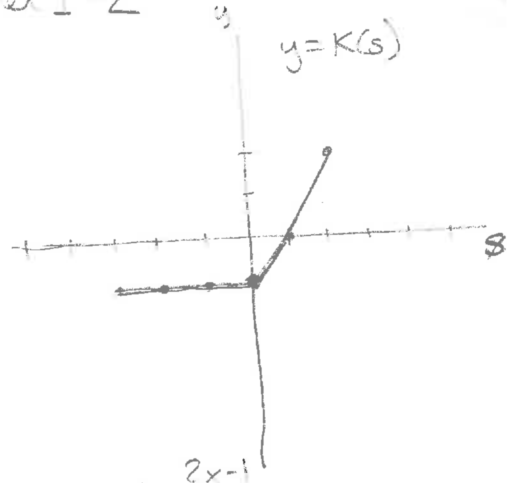
Review Solutions: Chapter 1-2

1) $g(x) = y$







3) $$\lim_{h\to 0} \frac{(x+h)-3(x+h)^2-x+3x^2}{h}$$

$$\lim_{h\to 0} \frac{(x+h)-3x^2-6xh-3h^2-\cancel{x}+3x^2}{h}$$

$$\lim_{h\to 0} 1-6x-3h = 1-6x$$

$$\boxed{f'(x) = 1-6x}$$

2) $24 = 4e^{2x-1}$

$6 = e^{2x-1}$

$\ln(6) = 2x-1$

$\ln(6)+1 = 2x$

$$\boxed{x = \frac{\ln 6+1}{2}}$$

4) $l_a(x) = f(a) + f'(a)(x-a)$

$a = 2$

$f(a) = \frac{1}{2}$

$f'(a) = -\frac{1}{4}$

$$\boxed{l_2(x) = \frac{1}{2} - \frac{1}{4}(x-2)}$$

5) $a = 3$

$f(3) = -1$

$f'(3) = 3^2-5 = 4$

$$\boxed{l_3(x) = -1+4(x-3)}$$

$l_3(3.1) = -1+4(3.1-3)$

$= -1+4(.1) = -1+.4 = \boxed{-.6 \approx f(3.1)}$

6) i)

a) $g'(3) = 1 \Rightarrow f(1) = \boxed{1}$

b) $\lim_{x\to -1^-} f(x) = \boxed{-1}$

c) $\lim_{x\to -1^+} f(x) = \boxed{-1}$

d) $\lim_{x\to -1} f(x) = \boxed{-1}$

e) $f(-1) = \boxed{2}$

f) $\lim_{x\to 3^-} f(x) = \boxed{1}$

g) $\lim_{x\to 3^+} f(x) = \boxed{-1}$

h) $\lim_{x\to 3} f(x)$ $\boxed{\text{DNE}}$

i) $f(3) = \boxed{4}$

j) $f'(2) = \boxed{-1}$ slope of line

k) $\lim_{x\to 1^+} \frac{1}{f} = \frac{1}{\lim_{x\to 1^+} f} = \boxed{\frac{1}{3}}$

l) $\lim_{x\to -\infty} \frac{1}{f} = \frac{1}{\lim_{x\to -\infty} f} = \boxed{0}$

m) $f'(-1)$ $\boxed{\text{DNE}}$

n) $\lim_{x\to -1} f(x) \cdot \lim_{x\to -1} x^2 = (-1)(-1)^2 = \boxed{-1}$

o) $\lim_{x\to -1} h(x) = 2(-1)^3+1 = -1$

$\lim f(y) = \boxed{-1}$

6 ii) $\boxed{-1, 1, 3}$

iii) $\frac{5-2}{4-3} = \boxed{3}$

iv) draw a tangent line to f at $x=4$. slope is about $\boxed{2}$

7) a) $\lim_{x \to 1} \frac{(x+3)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}} = \lim_{x \to 1} \frac{x+3}{x+1} = \frac{4}{2} = \boxed{2}$

b) $\lim_{x \to 0} \frac{\frac{1}{x-3} + \frac{1}{3}}{x} = \lim_{x \to 0} \frac{\cancel{3}+x-\cancel{3}}{(x-3)(3)} \cdot \frac{1}{x} = \lim_{x \to 0} \frac{1}{(x-3)(3)} = \boxed{-\frac{1}{9}}$

c) $\lim_{x \to \infty} \frac{4x^2-x+3}{5+x-3x^2} = \boxed{-\frac{4}{3}}$ numerator and denominator have same highest power

d) $\lim_{x \to 1^+} \frac{1}{1-x} = \boxed{-\infty}$ since $\lim_{x \to 0} \frac{1}{x} = \infty$

and let $x=2 \Rightarrow \frac{1}{1-2} = -1$ negative

e) $\frac{|x-2|}{x-2} = \begin{cases} \frac{x-2}{x-2} & x-2 \geq 0 \\ \frac{-(x-2)}{x-2} & x-2 < 0 \end{cases}$

$= \begin{cases} 1 & x \geq 2 \\ -1 & x < 2 \end{cases} \Leftarrow \lim_{x \to 2^-} \frac{|x-2|}{x-2} = \boxed{-1}$

f) $-1 \leq \sin x \leq 1$

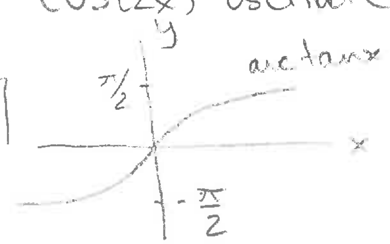
$-\frac{1}{x^2} \leq \frac{\sin x}{x^2} \leq \frac{1}{x^2}$

$\lim_{x \to \infty} -\frac{1}{x^2} \leq \lim_{x \to \infty} \frac{\sin x}{x^2} \leq \lim_{x \to \infty} \frac{1}{x^2}$

$0 \leq \lim_{x \to \infty} \frac{\sin x}{x^2} \leq 0$ $\quad \therefore \lim_{x \to \infty} \frac{\sin x}{x^2} = \boxed{0}$

g) $\lim_{x \to \infty} \cos(2x)$ DNE since $\cos(2x)$ oscillates

h) $2 \lim_{x \to -\infty} \arctan(x) = 2\left(-\frac{\pi}{2}\right) = \boxed{-\pi}$



i) $\lim_{y \to 0} \sin\left(\frac{1}{y}\right) = \lim_{t \to \infty} \sin(t)$ which is bounded by -1 and 1.

$\lim_{y \to 0} y^2 \sin\left(\frac{1}{y}\right) = \boxed{0}$

goes to 0

between -1 and 1

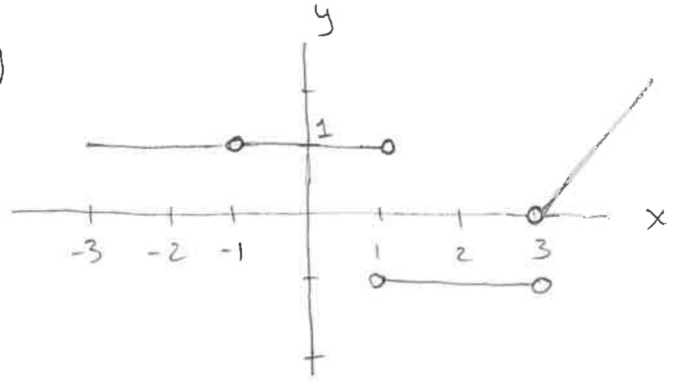
8) plug in 2 $c(2)^2 + 2(2) = 2^3 - c(2)$

$4c + 4 = 8 - 2c$

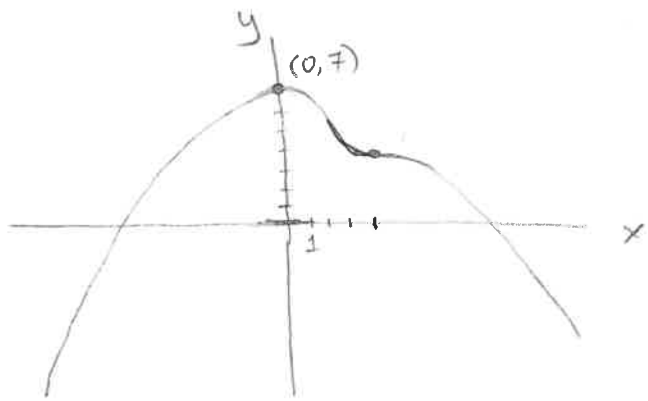
$6c = 4$

$\boxed{c = \frac{2}{3}}$

9)



10)



11) local min : 2, −4, 0 (y values)

local max : 8, 4 (y values)

global min : −4 (y values)

global max : DNE (y values

increasing : $(-\infty, -2]$, $[2, 6)$, $[8, 12]$

decreasing : $(-2, 2]$, $(6, 8]$

concave up : $(-2, 0)$ $(6, 12)$

concave down : $(0, 2)$