Power Rule

1. $f(x) = x - x^3$
2. $f(x) = \dfrac{4}{x^2} - \dfrac{x^2}{4}$
3. $h(x) = \dfrac{3}{\sqrt{x}}$
4. $f(x) = x^2 - e^2$
5. $g(x) = \sqrt{\sqrt{x}}$
6. $f(x) = \dfrac{x^2 - 1}{x}$
7. $f(x) = \dfrac{7x + 3x^2}{5\sqrt{x}}$

Chain Rule

Find the first derivative of the following functions:

8. $f(x) = (x^2 - 1)^{10}$
9. $f(x) = \sqrt{1 + \sqrt{1 + 2x}}$
10. $g(x) = (3x^2 + 3x - 6)^{-8}$
11. $f(x) = \sqrt[4]{9 - x}$

Product and Quotient Rule

Find the first derivative of the following functions:

12. $f(x) = (x + 1)(x^2 - 3)$ (Try this one in two different ways.)
13. $g(x) = \dfrac{3x^2 + 5x}{\sqrt{x}}$
14. $f(x) = x\sqrt{3x^2 - x}$
15. $f(x) = \dfrac{(5x^2 - 3)(x^2 - 2)}{x^2 + 2}$
16. $g(x) = \dfrac{x}{x + \frac{17}{x}}$
17. $h(x) = (\sqrt{x} - 4)^3(\sqrt{x} + 4)^5$

All Mixed Up: Power, Product, Quotient, Chain Rules

18. Find the first derivative of the following functions:

(a) $f(t) = 3t^2 + 2t$

(b) $g(w) = \dfrac{w^3}{(w+3)^5}$

(c) $h(s) = (s^{-2})^3$

(d) $f(x) = 5\sqrt{x}$ at 4

(e) $g(x) = \sqrt[3]{\sqrt[5]{\sqrt{x}}}$

(f) $f(x) = \pi^2$

(g) $m(t) = \sqrt{t^2 - 5t}$

(h) $g(y) = \sqrt{1 + \sqrt{1 + \sqrt{y}}}$

(i) $h(s) = (s+1)^5\sqrt{s-1}$

(j) $f(x) = \dfrac{2x-1}{\sqrt{x+1}}$

(k) $f(x) = \dfrac{(x+2)^2(3x - 4x^5)^{100}}{(8-x)^7}$

Derivatives of Exponential Functions

19. Find the first derivative of the following functions:

(a) $f(t) = e^{3t}$

(b) $g(z) = \left(\dfrac{2}{3}\right)^{3z-z^2}$

(c) $h(k) = 7e^{-5} - 7e^{-5k} + k^2 \ln(e^4)$

(d) $i(r) = 2^{4\sqrt{r}}$

(e) $A(t) = Pe^{rt}$ where P, r are constants

Derivatives of Logarithmic Functions

20. Find the first derivative of each function.

(a) $l(t) = \ln(x^2 - 1)$

(b) $h(x) = \ln(x^x)$

(c) $t(y) = y \ln \frac{1}{y}$

(d) $j(x) = \ln\left(\dfrac{(4x-1)^8(3x^2+14)^7}{\sqrt{x^2-4}}\right)$

(e) $k(s) = \log_2((5s^8 - 11)^3)$

Derivatives of Trig / Inverse Trig / Inverse Functions

21. Find the first derivative of each function

    (a) $a(s) = 2\sin^2(s) + 2\cos^2(s)$
    Do this one two ways.

    (b) $d(v) = \arccos(\cos(v))$
    Do this one two ways.

    (c) $b(t) = 4\ln(5\cos(t))$

    (d) $c(u) = \cos(\sin(u))$

    (e) $f(w) = \tan(w^2 + 1)$

    (f) $g(v) = \arcsin(\cos(v)) + \cos(\arcsin(v))$ and simplify your result

    (g) $h(y) = y^2 \arctan(4y)$

    (h) $i(z) = \sec^7(2z)$

22. If $f(2) = 4$ and $f'(2) = 7$ determine the derivative of f^{-1} at 4.

23. If $f(x) = \dfrac{2x-1}{3x+4}$ determine $\dfrac{d}{dx}f^{-1}(x)$

    (a) using the result relating f' and $(f^{-1})'$ obtained in class.

    (b) by determining f^{-1} and then differentiating.

All Mixed Up: Derivatives of Exponentials, Logarithms, Trig, Misc. Functions

24. Find the first derivative of the following functions:

    (a) $f(t) = 3e^{4t}$

    (b) $g(w) = \dfrac{e^3}{(3)^w}$

    (c) $h(s) = e^{2s}\ln(2s)$ at $1/2$

    (d) $f(x) = 5\sqrt{\log_3(x)}$

    (e) $g(x) = x^2 e^{x^2}$

    (f) $f(x) = x^e$

    (g) $f(x) = (\pi e)^2$

    (h) $m(t) = \tan(3t)$

    (i) $g(y) = y\cos(\ln y))$

    (j) $h(s) = s\sin s$

    (k) $f(x) = \dfrac{x}{\sin x}$

    (l) $f(x) = \dfrac{(x+2)^2(e)^{100+x^3}}{\sin^7(x)}$

Some "log trick" problems

25. Using the "log trick" show that $\dfrac{d}{dx}a^x = a^x\ln(a)$

26. Use the "log trick" to show that $(x^x)' = x^x(\ln(x) + 1)$

27. Use the "log trick" and the previous problem to determine $\dfrac{d}{dx}x^{x^x}$.

Miscillanious problems

28. If $g(d) = ab^2 + 3c^3d + 5b^2c^2d^2$, then what is $g''(d)$?

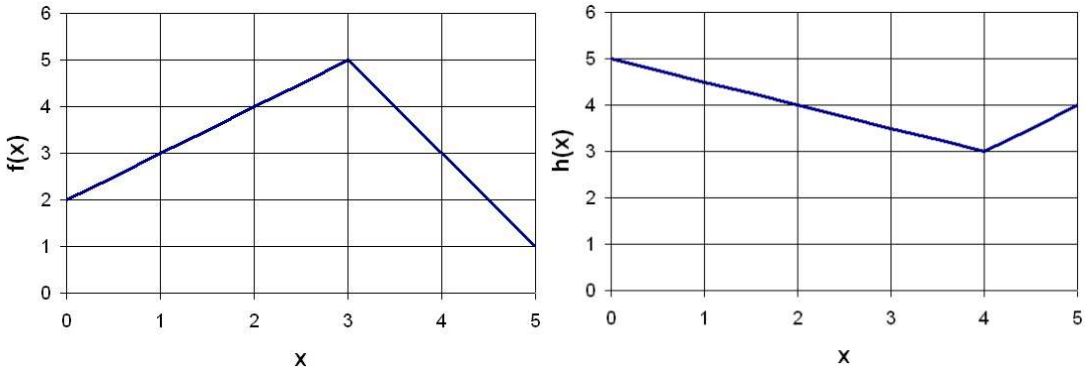
29. If $\frac{dy}{dx} = 5$ and $\frac{dx}{dt} = -2$ then what is $\frac{dy}{dt}$?

30. A ball is thrown into the air and its height h (in meters) after t seconds is given by the function $h(t) = 10 + 20t - 5t^2$. When the ball reaches its maximum height, its velocity will be zero.

    (a) At what time will the ball reach its maximum height?

    (b) What is the maximum height of the ball?

31. Given the graphs of $f(x)$ and $h(x)$.



    (a) The function $g = 10fh$. What is $g'(2)$?

    (b) The function $g = 10f(h)$. What is $g'(2)$?

    (c) The function $g = 10\frac{f}{h}$. What is $g'(2)$?

32. What is the line tangent to $f(x) = x^3$ at 2?

33. Find the derivative in $f(x) = \frac{x}{\sqrt{x}}$ in three ways. i) using algebra and the power rule, ii) the product rule and iii) the quotient rule. Carry through algebra to show that these are all equal.

34. Let $f(3) = 2$, $f'(3) = 4$, $g(3) = 1$, $g'(3) = 3$ and $f'(1) = 5$.

    (a) If $h(x) = f(x)g(x)$, what is $h'(3)$?

    (b) If $h(x) = \frac{f(x)}{g(x)}$, what is $h'(3)$?

    (c) If $h(x) = f \circ g(x)$, what is $h'(3)$?

35. A function has a local minimum at $x = -1$ and $x = 3$ and a local max at $x = 2$. What is a possible function for $f'(x)$?

36. If $u = ve^w + xy^v$, then what is $\frac{du}{dv}$?

37. Use the product rule to show that the derivative of $\tan(x)$ is $\sec^2(x)$.

38. For what value of x is $\frac{d}{dx}e^x$ equal to 1?

39. What is the line tangent to $f(x) = 2e^x$ at 1?

40. If $\ln(x) - y = 0$, find $\dfrac{dx}{dy}$.

41. Let $f(x) = e^{x^2}\cos(2x)\sqrt{3x+1}$, find $f'(x)$.

42. Let $f(x) = \dfrac{x^3}{3} + x^2 - 3x$ for all $x \in \mathbf{R}$.

    (a) For what values (there are two of them) is $f'(x) = 0$.

    (b) List the intervals where f is increasing. Don't use a graph.

    (c) List the intervals where f is decreasing. Don't use a graph.

    (d) Where does f have a local maximum?

    (e) What is the local minimum value of f?