Chapter 4 Review

1) a) $\lim_{x\to 0} \frac{x}{e^{4x}-1}$ $\frac{0}{1-1}$ "$\frac{0}{0}$"

$\lim_{x\to 0} \frac{1}{4e^{4x}} = \boxed{\frac{1}{4}}$

b) $\lim_{x\to 0} \frac{\sin(3x)}{\sin(5x)}$ "$\frac{0}{0}$"

$\lim_{x\to 0} \frac{3\cos(3x)}{5\cos(5x)} = \boxed{\frac{3}{5}}$

c) $\lim_{x\to 2} \frac{2e^{x-2}-x}{x^2-4}$ $\frac{2-2}{4-4}$ "$\frac{0}{0}$"

$\lim_{x\to 2} \frac{2e^{x-2}-1}{2x} = \frac{2-1}{2\cdot 2} = \boxed{\frac{1}{4}}$

d) $\lim_{x\to\infty} x^2 e^{-x^2}$ "$\infty\cdot 0$"

$\lim_{x\to\infty} \frac{x^2}{e^{x^2}}$ "$\frac{\infty}{\infty}$"

$\lim_{x\to\infty} \frac{2x}{2xe^{x^2}} = \lim_{x\to\infty} \frac{1}{e^{x^2}} = \boxed{0}$

e) $\lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{\sin(x)}\right)$ "$\infty - \infty$"

$\lim_{x\to 0^+} \frac{\sin(x)-x}{x\sin(x)}$ "$\frac{0}{0}$"

$\lim_{x\to 0^+} \frac{\cos(x)-1}{\sin(x)+x\cos(x)}$ $\frac{1-1}{0+0}$ "$\frac{0}{0}$"

$\lim_{x\to 0^+} \frac{-\sin(x)}{\cos x + \cos(x) - x\sin x} = \frac{0}{1+1-0} = \boxed{0}$

f) $\lim_{x\to 0} \frac{4x^3}{e^x} = \frac{0}{1} = \boxed{0}$

g) $\lim_{x\to\infty} (\ln(x))^{\frac{1}{x}}$ "∞^0"

$\ln((\ln x)^{\frac{1}{x}}) = \frac{1}{x}\ln(\ln x)$

$\lim_{x\to\infty} \frac{\ln(\ln(x))}{x}$ "$\frac{\infty}{\infty}$"

$\lim_{x\to\infty} \frac{\frac{1}{\ln x}\cdot\frac{1}{x}}{1} = \frac{0\cdot 0}{1} = 0$

$e^0 = \boxed{1}$

h) $\lim_{x\to 0^+} x\ln x$ "$0\cdot(-\infty)$"

$\lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}}$ "$\frac{-\infty}{\infty}$"

$\lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x\to 0^+} \frac{1}{x}\cdot\frac{-x^2}{1} = \boxed{0}$

i) $\lim_{x\to 0} \sin(x)^{\frac{1}{x}}$ "0^∞"

$\ln(\sin x)^{\frac{1}{x}} = \frac{1}{x}\ln(\sin x)$

$\lim_{x\to 0} \frac{1}{x}\ln(\sin x) \Rightarrow \infty\cdot(-\infty) = -\infty$

$e^{-\infty} = \boxed{0}$

j) $\lim_{x\to 0^+} \frac{\sin x}{\ln x}$ "$\frac{0}{\infty}$"

$\lim_{x\to 0^+} \frac{1}{\ln x}\cdot\sin x = 0\ 0 = \boxed{0}$

2) a) $f'(x) = 6t^2+6t+6 = 0$

$t^2+t+1 = 0$

$t = \frac{-1\pm\sqrt{1-4(1)(1)}}{2}$ imaginary

no critical points

b) $g'(r) = \frac{(r^2+1) - r(2r)}{(r^2+1)^2} = 0$

$-r^2+1 = 0$

$r^2 = 1$

$\boxed{r = \pm 1}$

2c) $h(x) = \sqrt{x} - x^{3/2}$

$h'(x) = \frac{1}{2\sqrt{x}} - \frac{3}{2}\sqrt{x} = 0$ multiply bothsides by $\sqrt{x}$

$\boxed{x=0}$

$\frac{1}{2} - \frac{3}{2}x = 0$

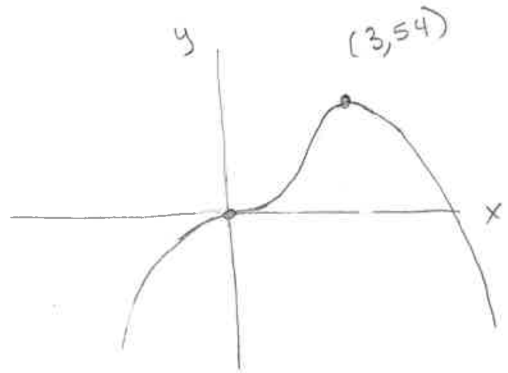
$\frac{1}{2} = \frac{3}{2}x \Rightarrow \boxed{x = \frac{1}{3}}$

d) $f'(\theta) = 2\sin(2\theta)\cos(2\theta)\cdot 2 = 0$

$\sin(2\theta) = 0 \Rightarrow \boxed{\theta = \frac{n\pi}{2},\ n \in \mathbb{Z}}$

$\cos(2\theta) = 0 \Rightarrow 2\theta = \frac{2n+1}{2} \Rightarrow \boxed{\theta = \frac{2n+1}{4} \quad n \in \mathbb{Z}}$

3) a)



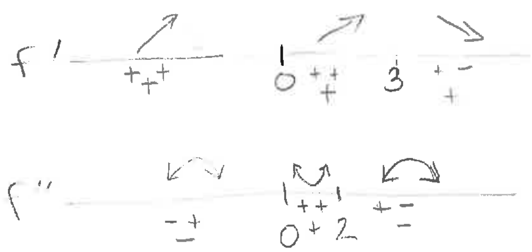
$f'(x) = 24x^2 - 8x^3 = 0$

$8x^2(3-x) = 0$

$x=0 \quad x=3$

$f''(x) = 48x - 24x^2 = 0$

$24x(2-x) = 0 \quad x=0,\ x=2$



$f(3) = 8\cdot 27 - 2\cdot 3\cdot 27 = 27(8-6) = 54$

$f(0) = 0$

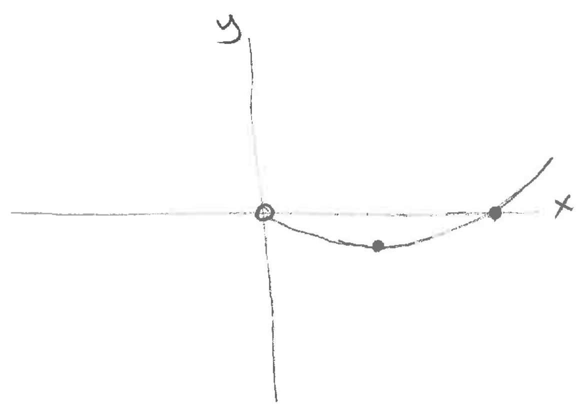
$g(x) = x\ln(x) \quad x > 0$

$g'(x) = \ln x + 1 = 0$

$\ln x = -1$

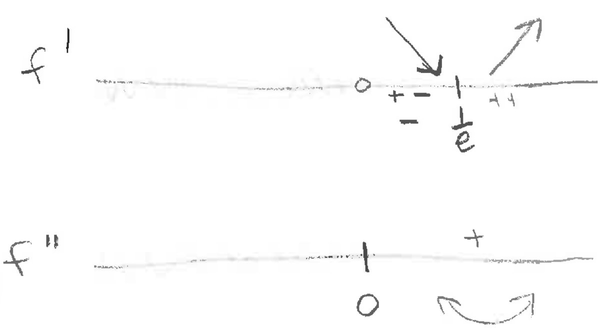
$x = e^{-1} = \frac{1}{e}$

b)



$g''(x) = \frac{1}{x} \neq 0$

$x \neq 0$



$g(1) = 0$

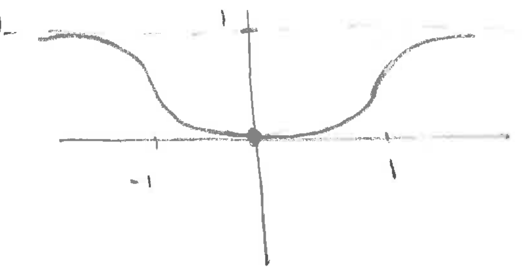
$\lim_{x \to 0^+} x\ln(x) = 0(-\infty)$

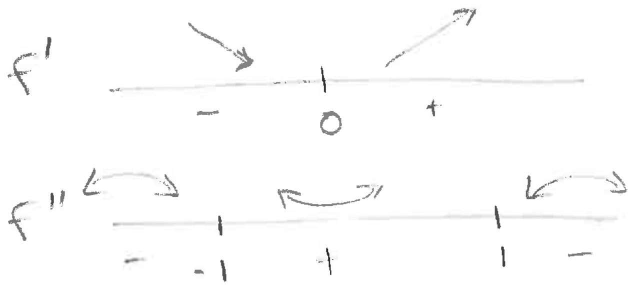
$\lim_{x \to 0^+} \frac{\ln x}{\frac{1}{x}} = \frac{-\infty}{\infty}$

$\lim_{x \to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = 0$

$g\left(\frac{1}{e}\right) = \frac{1}{e}\ln(e^{-1}) = -\frac{1}{e}$

c)





$$h(x) = \frac{x^2}{x^2+3}$$

$$h'(x) = \frac{2x(x^2+3) - x^2(2x)}{(x^2+3)^2} = \frac{6x}{(x^2+3)^2} = 0$$

$x = 0$

$$h''(x) = \frac{6(x^2+3)^2 - 6x \cdot 2(x^2+3)2x}{(x^2+3)^4}$$

$$= \frac{6(x^2+3)(x^2+3-4x^2)}{(x^2+3)^4} = 0$$

$3(-x^2+1) = 0 \quad x = \pm 1$

$\lim_{x \to \infty} \frac{x^2}{x^2+3} = 1 \qquad h(0) = \frac{0}{3} = 0$

$\lim_{x \to -\infty} \frac{x^2}{x^2+3} = 1$

4) a) $\frac{1}{2\sqrt{xy}}\left(y + x\frac{dy}{dx}\right) = 2xy + x^2\frac{dy}{dx}$

$$\frac{y}{2\sqrt{xy}} - 2xy = x^2\frac{dy}{dx} - \frac{x}{2\sqrt{xy}}\frac{dy}{dx}$$

$$\boxed{\frac{dy}{dx} = \frac{\frac{y}{2\sqrt{xy}} - 2xy}{x^2 - \frac{x}{2\sqrt{xy}}}}$$

b) $\sin(x-y)\left(1 - \frac{dy}{dx}\right) = \frac{dy}{dx}\sin x + y\cos x$

$$\sin(x-y) - y\cos x = \frac{dy}{dx}(\sin x + \sin(x-y))$$

$$\boxed{\frac{dy}{dx} = \frac{\sin(x-y) - y\cos x}{\sin x + \sin(x-y)}}$$

c) $3x^2 + 2xy + x^2\frac{dy}{dx} + 8y\frac{dy}{dx} = 0$

$$\boxed{\frac{dy}{dx} = \frac{-3x^2 - 2xy}{x^2 + 8y}}$$

d) $2x\sin y + x^2\cos y\frac{dy}{dx} = \frac{1}{xy}\left(y + x\frac{dy}{dx}\right)$

$$2x\sin y - \frac{1}{x} = \frac{dy}{dx}\left(\frac{1}{y} - x^2\cos y\right)$$

$$\boxed{\frac{dy}{dx} = \frac{2x\sin y - \frac{1}{x}}{\frac{1}{y} - x^2\cos y}}$$

5) $\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = \frac{-y^{1/3}}{x^{1/3}} \text{ at } (-3\sqrt{3}, 1) = \frac{-1^{1/3}}{(-3\sqrt{3})^{1/3}} = \frac{1}{(3\sqrt{3})^{1/3}}$$

$$y - 1 = \frac{1}{(3\sqrt{3})^{\frac{1}{3}}}(x - (-3\sqrt{3}))$$

$$\boxed{y - 1 = \frac{1}{(3\sqrt{3})^{\frac{1}{3}}}(x + 3\sqrt{3})}$$

e) $\cosh(4x^2)8x = -\sinh(7y)7\frac{dy}{dx}$

$$\boxed{\frac{dy}{dx} = -\frac{8x\cosh(4x^2)}{7\sinh(7y)}}$$

$x^2 + 2y^2 = 6$

6) $2x + 4y \frac{dy}{dx} = 0$

$2x + 4y(1) = 0$

$y = -\frac{1}{2}x$

$x^2 + \frac{1}{2}x^2 = 6$

$\frac{3}{2}x^2 = 6$

$x = \pm 2$

$\boxed{(2,-1),\ (-2,1)}$

7) $h'(3) = 0$

$h'(x) = 2(t^3 - b)(3t^2)$

$h'(3) = 2(27 - b)(3 \cdot 9) = 0$

$\boxed{b = 27}$

critical points are $x = 3$ and $x = 0$

8) $f'(x) = -\sin x - 1 = 0$

$\sin x = -1$

$x = \frac{3\pi}{2}$

$\boxed{f(0) = 1 \quad \text{max}}$

$f\left(\frac{3\pi}{2}\right) = -\frac{3\pi}{2} \approx -4.7$

$\boxed{f(2\pi) = 1 - 2\pi \approx -5.3 \quad \text{min}}$

9) $g'(x) = 3x^2 - 3 = 0$

$x^2 = 1 \Rightarrow x = \pm 1$

(-1 not in interval)

$f(0) = 1$

$\boxed{f(1) = -1 \quad \text{min}}$

$\boxed{f(3) = 19 \quad \text{max}}$

10) $x + y = 23 \quad y = 23 - x$

$p = x \cdot y$

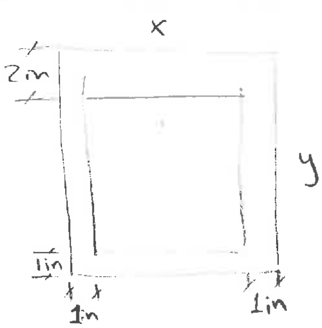
$p = x(23 - x) = 23x - x^2$

$p' = 23 - 2x = 0$

$x = \frac{23}{2} \Rightarrow y = 23 - \frac{23}{2} = \frac{23}{2}$

$\boxed{x = \frac{23}{2},\quad y = \frac{23}{2}}$

11)



$A = xy = 180$

$y = \frac{180}{x}$

Printed area $= (x-2)(y-3)$

$PA = (x-2)\left(\frac{180}{x} - 3\right)$

$= 180 - \frac{360}{x} - 3x + 6$

$= 186 - \frac{360}{x} - 3x$

$PA' = \frac{360}{x^2} - 3 = 0$

$\frac{360}{x^2} = 3$

$x^2 = 120$

$x = \pm\sqrt{120} \approx 10.95 \text{ in}$

$y = \frac{180}{\sqrt{120}} \approx 16.44 \text{ in}$

12) profit = (price − cost)(# of bulbs)   cost per bulb = \$4

profit = $(10-4+x)(40-3x)$ $40-3x \geq 0$ $40 \geq 3x$ $13.3 \geq x$

for every ↑ in bulb x the company adds \$1 but sells 3 less

$$P = 240 + 22x - 3x^2$$

$$P' = 22 - 6x = 0$$

$$x = \frac{22}{6} = \frac{11}{3}$$ can't have $3.\overline{6}$ of a bulb so lets try 3 and 4

profit = $(6+\frac{11}{3})(40-11)$
\$ 280.33

profit(3) = (9)(31) = \$279

profit(4) = (10)(28) = \$280

\$280.33 is biggest so sell for $10+3.67 = \boxed{\$13.67}$

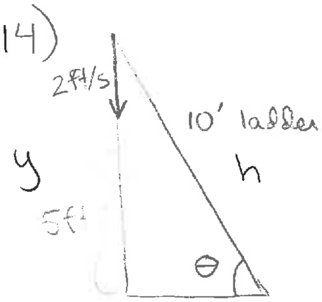
13) Vol. of sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 = \frac{\pi D^3}{6}$

$$V = \frac{\pi D^3}{6}$$ $r=2 \Rightarrow d=4$

$$\frac{dV}{dt} = \frac{dV}{dD}\cdot\frac{dD}{dt} = \frac{3\pi D^2}{6}\cdot\frac{dD}{dt}$$

$$\frac{dD}{dt} = \frac{\frac{dV}{dt}}{\frac{3\pi D^2}{6}} = \frac{-3}{\frac{3\pi(4)^2}{6}} = -\cancel{3}\cdot\frac{\overset{3}{\cancel{6}}}{\cancel{3}\pi\,\underset{2}{\cancel{4}}\cdot 4} = \boxed{-\frac{3}{8\pi}\ \frac{in}{sec}}$$

14)



$$\sin\theta = \frac{y}{h}$$

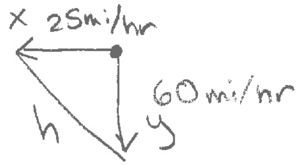
$$y = 10\sin\theta$$

$$\frac{dy}{dt} = 10\cos\theta\cdot\frac{d\theta}{dt}$$

$$-2 = 10\left(\frac{5}{10}\right)\frac{d\theta}{dt}$$

$$\boxed{\frac{d\theta}{dt} = -\frac{2}{5}\ ft/s}$$

15)



$$x^2+y^2=h^2$$

distance = rate · time

$x = 25t \Rightarrow 50$

$y = 60t \Rightarrow 120$ at 2 hrs

$$\cancel{2}x\frac{dx}{dt}+\cancel{2}y\frac{dy}{dt}=\cancel{2}h\frac{dh}{dt}$$

$$50\cdot 25+120\cdot 60=\sqrt{50^2+120^2}\,\frac{dh}{dt}$$

$$\frac{1250+7200}{\sqrt{16{,}900}}=\frac{dh}{dt}$$

$$\frac{dh}{dt}=\frac{8450}{130}=\boxed{65\text{ mph}}$$

16

		-4		-1		0		3	
f		inf. pt		local min		inf. pt		local max	
f'	−	−	−	0	+	+	+	0	−
f''	−	0	+	+	+	0	−	−	−