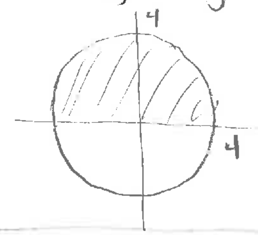
Chapter 5 Review

1) a) $y = \sqrt{16-x^2}$

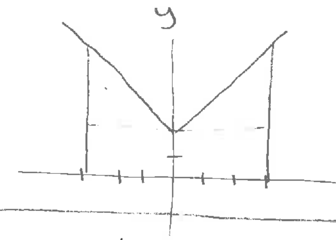
$y^2 = 16 - x^2$

$x^2 + y^2 = 16$



$\frac{1}{2}\pi(4)^2 = \boxed{8\pi}$

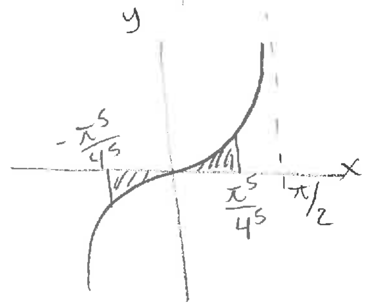
b)



$2(6) + 2(\tfrac{1}{2})3\cdot 3$

$12 + 9 = \boxed{21}$

c)

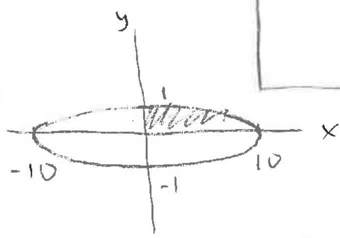


$\boxed{0}$ same area above and below graph

d) $y = \sqrt{1 - \frac{x^2}{100}}$

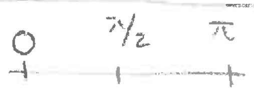
$y^2 = 1 - \frac{x^2}{100}$

$\frac{x^2}{100} + y^2 = 1$



$\frac{\pi(10)(1)}{4} = \frac{10\pi}{4} = \boxed{\frac{5\pi}{2}}$

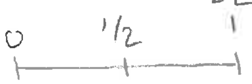
2a)



$L_0^{\pi/2}(\cos^2(x), 2) = \frac{\pi}{2}\left[\cos^2 0 + \cos^2 \frac{\pi}{2}\right] = \frac{\pi}{2}(1^2 + 0) = \boxed{\frac{\pi}{2}}$

$R_0^{\pi/2}(\cos^2(x), 2) = \frac{\pi}{2}\left[\cos^2 \frac{\pi}{2} + \cos^2 \pi\right] = \frac{\pi}{2}(0 + (-1)^2) = \boxed{\frac{\pi}{2}}$

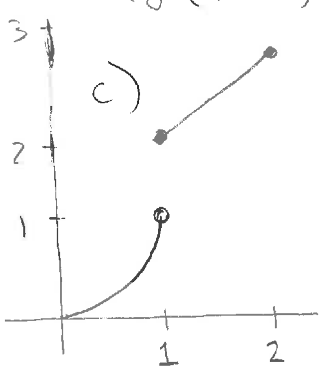
b)



$L_0^1(x^2+2, 2) = \frac{1}{2}\left[(0^2+2) + \left(\left(\tfrac{1}{2}\right)^2+2\right)\right] = \frac{1}{2}\left[4 + \frac{1}{4}\right] = \boxed{\frac{17}{8}}$

$R_0^1(x^2+2, 2) = \frac{1}{2}\left[\left(\left(\tfrac{1}{2}\right)^2+2\right) + 1^2+2\right] = \frac{1}{2}\left[\frac{1}{4} + 5\right] = \boxed{\frac{21}{8}}$

c)



Left sum $= 1[0 + 2] = \boxed{2}$

Right sum $= 1[2 + 3] = \boxed{5}$

3) $\int_a^b f'(x)\,dx = f(b) - f(a)$

signed area of graph

a) $\int_{-3}^{0} f'(x)\,dx = f(0) - f(-3)$

$-\left((1)(1) - \frac{1}{4}\pi(1)^2\right) + \frac{1}{2}(2)(3) = -3 - f(-3)$

$f(-3) = -3 + 1 - \frac{\pi}{4} - 3 = \boxed{-\frac{\pi}{4} - 5}$

b) $\int_{-2}^{0} f'(x)\,dx = f(0) - f(-2)$

$\frac{1}{2}(2)(3) = -3 - f(-2)$

$f(-2) = -3 - 3 = \boxed{-6}$

c) $\int_{-1}^{0} f'(x)\,dx = f(0) - f(-1)$

$\frac{1}{2}(2)(3) - \frac{1}{2}(1)\left(\frac{3}{2}\right) = -3 - f(-1)$

$f(-1) = \underset{-\frac{24}{4}}{-3-3} + \frac{3}{4} = \boxed{-\frac{21}{4}}$

d) $\int_{0}^{1} f'(x)\,dx = f(1) - f(0)$

$\frac{1}{2}(1)(3) = f(1) - (-3)$

$f(1) = -3 + \frac{3}{2} = \boxed{-\frac{3}{2}}$

e) $\int_{0}^{3} f'(x)\,dx = f(3) - f(0)$

$\frac{1}{2}(1)(3) - \frac{1}{2}\pi(1^2) = f(3) - (-3)$

$f(3) = \frac{3}{2} - \frac{\pi}{2} - 3 = \boxed{-\frac{3}{2} - \frac{\pi}{2}}$

4) $\int \sin x\,dx = -\cos x + C_1$

@ π : $-\cos(\pi) + C_1 = 2$

$-(-1) + C_1 = 2 \qquad C_1 = 1$

$\int x - 2\pi\,dx = \frac{x^2}{2} - 2\pi x + C_2$

for continuity

$-\cos x + C_1 = \frac{x^2}{2} - 2\pi x + C_2$ at $x = 2\pi$

$-\cos(2\pi) + 1 = \frac{(2\pi)^2}{2} - 2\pi(2\pi) + C_2$

$-1 + 1 = 2\pi^2 - 4\pi^2 + C_2 \Rightarrow C_2 = 2\pi^2$

$$\boxed{F(x) = \begin{cases} -\cos(x) + 1 \\ \frac{x^2}{2} - 2\pi x + 2\pi^2 \end{cases}}$$

5) $v(t) = -t+4$ at $t=0,\ x=0$

$x(t) = \int v(t)\,dt = \int -t+4\,dt = -\frac{t^2}{2} + 4t + C$

$0 = -\frac{0^2}{2} + 4(0) + C \quad C = 0$

$x(t) = -\frac{t^2}{2} + 4t \Rightarrow x(6) = -\frac{6^2}{2} + 4(6) = \boxed{6\text{ miles}}$

6) a) $\int \sqrt{x}\,dx = \frac{2}{3}x^{3/2} + C$

$0 = \frac{2}{3}4^{3/2} + C$

$C = -\frac{2}{3}(2)^3 = -\frac{16}{3}$

$\boxed{F(x) = \frac{2}{3}x^{3/2} - \frac{16}{3}}$

b) $f'(x) = \int x^2+4\,dx = \frac{x^3}{3} + 4x + C$

$f'(3) = \frac{3^3}{3} + 4(3) + C = 1 \quad C = -20$

$f(x) = \int \frac{x^3}{3} + 4x - 20\,dx = \frac{x^4}{12} + 2x^2 - 20x + C_2$

$f(1) = \frac{1}{12} + 2 - 20 + C_2 = 6 \quad C_2 = 24 - \frac{1}{12} = \frac{287}{12}$

$\boxed{f(x) = \frac{x^4}{12} + 2x^2 - 20x + \frac{287}{12}}$

7) a) $f'(x) = \frac{d}{dx}\int_x^e \sin(t^2)\,dt = \frac{d}{dx}\left(-\int_e^x \sin(t^2)\,dt\right) = \boxed{-\sin x^2}$

b) $g'(y) = \frac{d}{dy}\int_1^{y^2}(x^2+1)^3\,dx = ((y^2)^2+1)^3 \cdot 2y = \boxed{(y^4+1)^3(2y)}$

c) $h'(x) = \frac{d}{dx}\int_{-x}^{x} 2^{s^s}\,ds = \frac{d}{dx}\left[-\int_1^{-x} 2^{s^s}\,ds + \int_1^{x} 2^{s^s}\,ds\right]$

$= -2^{(-x)^{(-x)}}(-1) + 2^{x^x} = \boxed{2^{(-x)^{(-x)}} + 2^{x^x}}$

8) a) $\int_0^{11} f = \int_0^1 f + \int_1^{11} f$

$29 = -7 + \int_1^{11} f$

$\int_1^{11} f = \boxed{36}$

b) $\int_0^{10} f = \int_0^4 f + \int_4^{10} f$

$\int_4^{10} f = -\int_{10}^4 f$
-3
$= 3$

$8 = \int_0^4 f + 3$

$\int_0^4 f = 5$

$\int_0^4 5f(x) + \sqrt{x}\,dx = 5\int_0^4 f(x)\,dx + \int_0^4 \sqrt{x}\,dx$

$= 5(5) + \frac{2}{3}x^{3/2}\Big|_0^4$

$= 25 + \frac{2}{3}(4)^{3/2} - 0 = 25 + \frac{16}{3} = \boxed{\frac{91}{3}}$

9) $\int_a^{2a} \frac{3}{4} x (x^2-a^2)^2 \, dx$

$u = x^2 - a^2$

$du = 2x\,dx$

$\int \frac{3}{4} \cdot \frac{1}{2} u^2 \, du = \frac{u^3}{8}$

$\left. \frac{(x^2-a^2)^3}{8} \right|_a^{2a} = \frac{(3a^2)^3}{8} - 0 = 1$

$(3a^2)^3 = 8$

$3a^2 = 2$

$a^2 = \frac{2}{3} \Rightarrow a = \pm\sqrt{\frac{2}{3}}$ $\quad \boxed{a = \sqrt{\frac{2}{3}}}$

10) a) $\boxed{\frac{4x^3}{3} - \frac{5}{2}x^2 + 3x + C}$

b) $\int 5x^5 - 2x^6 + 3x^7 \, dx$

$= \boxed{\frac{5}{6}x^6 - \frac{2}{7}x^7 + \frac{3}{8}x^8 + C}$

c) $\int 4x^{-1/2} + 5x \, dx$

$= 8x^{\frac{1}{2}} + \frac{5}{2}x^2 + C$

$= \boxed{8\sqrt{x} + \frac{5}{2}x^2 + C}$

d) $\left. 5x - x^2 + x^3 \right|_1^4$

$= 20 - 16 + 64 - 5 + 1 - 1 = \boxed{63}$

e) $\left. -3\cos(x) \right|_0^{\pi}$

$-3\cos(\pi) + 3\cos(0)$

$-3(-1) + 3(1) = \boxed{6}$

f) $\int \frac{x^2}{\sqrt{x^3+5}} \, dx$ $\quad u = x^3 + 5$

$du = 3x^2\,dx$

$\frac{1}{3} \int u^{-1/2} \, du$

$\frac{2}{3} u^{1/2} \Rightarrow \boxed{\frac{2}{3}\sqrt{x^3+5} + C}$

g) $\int \sin^3 x \cos x \, dx$ $\quad u = \sin x$

$du = \cos x \, dx$

$\int u^3 \, du = \frac{u^4}{4}$

$\boxed{\frac{\sin^4 x}{4} + C}$

h) $\int \sec^2(x) \tan(x) \, dx$

$u = \tan x$

$du = \sec^2 x \, dx$

$\int u \, du = \frac{u^2}{2}$

$\boxed{\frac{\tan^2 x}{2} + C}$

10) i) $\int \frac{x}{\sqrt{1-x}}\,dx$ $\quad u = 1-x \Rightarrow x = 1-u$, $du = -dx$

$\int \frac{1-u}{\sqrt{u}}(-du) = \int -u^{-1/2} + u^{1/2}\,du$

$= -2u^{1/2} + \frac{2}{3}u^{3/2}$

$\Rightarrow \boxed{-2\sqrt{1-x} + \frac{2}{3}(1-x)^{3/2} + C}$

k) $\int_1^9 \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$ $\quad u = \sqrt{x}$, $du = \frac{1}{2\sqrt{x}}\,dx$

$\int 2\cos u\,du = 2\sin u$

$\Rightarrow 2\sin\sqrt{x}\Big|_1^9 = \boxed{2\sin(3) - 2\sin(1)}$

l) $\int \frac{(\ln x)^3}{x}\,dx$ $\quad u = \ln x$, $du = \frac{1}{x}\,dx$

$\int u^3\,du = \frac{u^4}{4}$

$\Rightarrow \boxed{\frac{(\ln x)^4}{4} + C}$

h) $\int_0^{\pi/2} \frac{\sin(x)}{1+\cos(x)}\,dx$ $\quad u = 1+\cos x$, $du = -\sin x\,dx$

$\int -\frac{1}{u}\,du = -\ln|u|$

$\Rightarrow -\ln|1+\cos(x)|\Big|_0^{\pi/2}$

$= -\ln\left|1+\cos\left(\frac{\pi}{2}\right)\right| + \ln|1+\cos(0)|$

$= -\ln|1| + \ln|2| = \boxed{\ln 2}$

) $\int \frac{1}{9x^2+1}\,dx = \boxed{\frac{\tan^{-1}(3x)}{3} + C}$

j) $\int_0^2 x(5-x^2)^{3/2}\,dx$ $\quad u = 5-x^2$, $du = -2x\,dx$

$\int -\frac{1}{2}u^{3/2}\,du$

$-\frac{1}{2}\cdot\frac{2}{5}u^{5/2}$

$\Rightarrow -\frac{1}{5}(5-x^2)^{5/2}\Big|_0^2$

$= -\frac{1}{5}(1)^{5/2} + \frac{1}{5}(5)^{5/2}$ $\quad 5^{5/2} = \sqrt{5}\cdot\sqrt{5}\cdot\sqrt{5}\cdot\sqrt{5}\cdot\sqrt{5}$

$= \boxed{-\frac{1}{5} + 5\sqrt{5}}$

m) $\int_0^{\pi} \sin x\, e^{\cos x}\,dx$ $\quad u = \cos x$, $du = -\sin x\,dx$

$\int -e^u\,du = -e^u$

$\Rightarrow -e^{\cos x}\Big|_0^{\pi}$

$= -e^{-1} + e^1 = \boxed{e - \frac{1}{e}}$

o) $\int \frac{x}{9x^2+1}\,dx$ $\quad u = 9x^2+1$, $du = 18x\,dx$

$\int \frac{1}{18}\frac{1}{u}\,du$

$= \frac{1}{18}\ln|u|$

$\Rightarrow \boxed{\frac{1}{18}\ln(9x^2+1) + C}$

q) $\int 9x + \frac{1}{x}\,dx$

$= \boxed{\frac{9}{2}x^2 + \ln|x| + C}$

10) r) $\int \frac{x}{\sqrt{9x^2+1}}\,dx$ $u = 9x^2+1$ $du = 18x\,dx$

$\int \frac{1}{18} u^{-1/2}\,du = \frac{2}{18} u^{1/2}$

$\Rightarrow \boxed{\frac{1}{9}\sqrt{9x^2+1} + C}$

t) $\int \frac{1}{\sqrt{1-9x^2}}\,dx = \boxed{\frac{\sin^{-1} 3x}{3} + C}$

u) $\int \frac{e^{2x}}{\sqrt{1-e^{4x}}}\,dx$ $u = e^{2x}$ $du = 2e^{2x}\,dx$

$\int \frac{1}{2} \frac{1}{\sqrt{1-u^2}}\,du = \frac{1}{2}\sin^{-1} u$

$\Rightarrow \boxed{\frac{1}{2}\sin^{-1}(e^{2x}) + C}$

w) $u = x$ $dv = \cos x$

1 $+$ $\sin x$

0 $-$ $-\cos x$

$\boxed{x\sin x + \cos x + C}$

y) $u = x^2$ $+$ $dv = \sin x$

$2x$ $-$ $-\cos x$

2 $+$ $-\sin x$

0 $\cos x$

$\boxed{-x^2\cos x + 2x\sin x + 2\cos x + C}$

s) $\int x^3 e^{x^2}\,dx$ $u = x^2$ $du = 2x\,dx$

$\int x^2 \cdot x\, e^{x^2}\,dx$

$\int \frac{1}{2} u e^u\,du$

parts $\bar{u} = \frac{1}{2}u$ $v = e^u$

$\bar{u}' = \frac{1}{2}$ $dv = e^u$

$= \frac{1}{2}ue^u - \int \frac{1}{2}e^u$

$= \frac{1}{2}ue^u - \frac{1}{2}e^u \Rightarrow \boxed{\frac{x^2}{2}e^{x^2} - \frac{1}{2}e^{x^2} + C}$

v) $\int \frac{\arcsin(x)}{\sqrt{1-x^2}}\,dx$ $u = \arcsin x$ $du = \frac{1}{\sqrt{1-x^2}}\,dx$

$\int u\,du = \frac{u^2}{2}$

$\Rightarrow \boxed{\frac{(\arcsin x)^2}{2} + C}$

x) $\int \ln x\,dx$

$u = \ln x$ $v = x$

$du = \frac{1}{x}\,dx$ $dv = 1\,dx$

$= x\ln x - \int x \cdot \frac{1}{x}\,dx$

$= \boxed{x\ln x - x + C}$

z) $\int e^x \sin x\,dx$ $u = e^x$ $v = -\cos x$ $du = e^x\,dx$ $dv = \sin x\,dx$

$= -e^x\cos x + \int e^x\cos x\,dx$ $u = e^x$ $v = \sin x$ $du = e^x\,dx$ $dv = \cos x\,dx$

$\int e^x \sin x\,dx = -e^x\cos x + e^x\sin x - \int e^x\sin x\,dx$

$2\int e^x\sin x\,dx = -e^x\cos x + e^x\sin x$

$\int e^x\sin x\,dx = \boxed{\frac{1}{2}\left[-e^x\cos x + e^x\sin x\right] + C}$

11) $f(x) = \int_0^x \sin(t)\,dt$

$f'(x) = \frac{d}{dx}\int_0^x \sin(t)\,dt = \sin x = 0$

$x = 0,\ x = \pi,\ x = 2\pi$

$x=0:\ f(x) = \int_0^0 \sin(t)\,dt = 0$

$x=\pi:\ f(x) = \int_0^{\pi} \sin(t)\,dt = -\cos t\Big|_0^{\pi} = 1+1 = 2$

$x=2\pi:\ f(x) = \int_0^{2\pi} \sin(t)\,dt = -\cos t\Big|_0^{2\pi} = -1+1 = 0$

minimun is 0 at 0 and 2π
maximum is 2 at π

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12) a) False - can't multiply 2 signed areas to get a single signed area.

or $\int x\sin x\,dx \neq \frac{x^2}{2}(-\cos x)$

b) False - can't divide signed area

or $\int \frac{x^2+1}{x}\,dx \neq \frac{\frac{x^3}{3}+x}{\frac{x^2}{2}}$

c) False $\frac{1}{x}$ is not defined at 0

d) False not true when $n=-1$