# Indefinite Integrals

## Goal

In this section, we will introduce antiderivatives and indefinite integrals.

## Antiderivatives

Loosely speaking, antidifferentiation is the opposite of differentiation.

**Example 1.** Find a function whose derivative is $f(x) = 2x + 5$.

**Definition 2.** A function $F$ is an *antiderivative* of the function $f$ (on an interval $I$) if

$$F'(x) = f(x)$$

(on $I$) whenever $f$ is defined.

**Note 3.** Notice that we said *an* antiderivative and not *the* antiderivative. Why?

**Example 4.** Find an antiderivative of $f(x) = \frac{3}{\sqrt{1-x^2}}$.

**Example 5.** If $f(-1) = 2$ and $f'(x) = 5x^4 - 3x^2 + 4$, find $f$.

 

## Indefinite integrals

For a given continuous function $f$, there is an entire family of antiderivatives: $G(x) = F(x) + C$, where $F$ is any antiderivative of $f$. We call $C$ the *constant of integration*.

**The Picture:**

**Definition 6.** The collection of antiderivatives of a function $f$ is called the *indefinite integral of $f$ with respect to $x$* and is denoted by

$$\int f(x)\ dx = F(x) + C,$$

where $F'(x) = f(x)$.

**Note 7.** There is a very good reason why the notation for definite integrals (i.e., net signed area under a curve) is similar to the notation for the collection of antiderivatives. In the next section, we will make this connection explicit.

**Example 8.** Compute $\int 5x^4\ dx$.

**Theorem 9.** We have the following basic integral formulas.

| | |
|---|---|
| $\int f(x) \pm g(x)\ dx =$ ____________________ | $\int k \cdot f(x)\ dx =$ ____________________ |
| $\int x^n\ dx =$ ____________________ | $\int e^x\ dx =$ ____________________ |
| $\int \frac{1}{x}\ dx =$ ____________________ | $\int \sin(x)\ dx =$ ____________________ |
| $\int \cos(x)\ dx =$ ____________________ | $\int \sec^2(x)\ dx =$ ____________________ |
| $\int \sec(x)\tan(x)\ dx =$ ____________________ | $\int \frac{1}{\sqrt{1-x^2}}\ dx =$ ____________________ |
| $\int \frac{1}{1+x^2}\ dx =$ ____________________ | $\int \frac{1}{x\sqrt{x^2-1}}\ du =$ ____________________ |

**Important Note 10.**

1. Each of the above are easily proved by just differentiating the right hand side.
2. Warning:

$$\int f(x) \cdot g(x)\ dx \neq \int f(x)\ dx \cdot \int g(x)\ dx \qquad \text{and} \qquad \int \frac{f(x)}{g(x)}\ dx \neq \frac{\int f(x)\ dx}{\int g(x)\ dx}$$

**Example 11.** Compute each of the following indefinite integrals.

(a) $\int 5\ dx$

(b) $\int 2x^3 + x^2 - 5x + 5\ dx$

(c) $\int -2\sqrt{x}\ dx$

(d) $\displaystyle\int \frac{x+1}{\sqrt{x}}\,dx$

(e) $\displaystyle\int \frac{1}{x^3}\,dx$

(f) $\displaystyle\int \frac{x+5}{x^2}\,dx$

(g) $\displaystyle\int \frac{\sin(x)}{\cos^2(x)}\,dx$

 