Quiz 3

Your Name:

Instructions

This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized*. Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 8 points for a total of 32 points. Good luck and have fun!

Part A

Complete **two** of the following problems.

A1. Recent archaeological work on Mars discovered a site containing a pile of white spheres, each about the size of a tennis ball. A plaque near the mound states that each sphere contains a jewel that come in many different colors while strictly more than half of the spheres contain jewels of the same color. When two spheres are brought together, they both glow white if their internal jewels are the same color; otherwise, no glow. In how few tests can you find a sphere that you are certain holds a jewel of the majority color if the number of spheres in the pile is 6. You must provide sufficient justification. In particular, you need to argue that you cannot guarantee few tests.

A2. There is a plate of 40 cookies. You and your friend are going to take turns taking either 1 or 2 cookies from the plate. However, it is a faux pas to take the last cookie, so you want to make sure that you do not take the last cookie. How can you guarantee that you will never be the one taking the last cookie? You must justify your answer.

A3. My Uncle Robert owns a stable with 25 race horses. He wants to know which three are the fastest. He owns a race track that can accommodate five horses at a time. What is the minimum number of races required to determine the fastest three horses? *Note:* You do not need to justify that your solution attains a minimum.

Part B

Complete **two** of the following problems.

B1. Suppose we must place the letters A, B, C, D, E into the grid below, one per box, so that each row, each column, and each of the two long diagonals contain one of each letter. How many ways are there to fill out the grid and satisfy these conditions? You must justify your answer.

A				B
	B			
		C		
D				E

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B2. A soul swapping machine swaps the souls inside two bodies placed in the machine. Soon after the invention of the machine an unforeseen limitation is discovered: swapping only works on a pair of bodies once. Souls get more and more homesick as they spend time in another body and if a soul is not returned to its original body after a few days, it will kill its current host. Bart (B), Lisa (L), Homer (H), Marge (M), and Ned (N) were involved in a soul-swapping bonanza that resulted in Bart's soul being Lisa's body, Lisa's soul being in Homer's body, Homer's soul being in Marge's body, Marge's soul in Ned's body, and Ned's soul being in Bart's body. Thankfully, Krusty the Clown (K) and Santa's Little Helper (S) never utilized the machine and are available to help put everyone's soul back in the appropriate body. Find a way to return all the souls to their respective owners. You method must guarantee that pair of bodies never sat in the machine more than once.

B3. Consider the light-up squares problem that we encountered in Problems 31 and 32. In class we proved that it is not possible for a starting configuration with fewer than n initial lit squares to result in the entire $n \times n$ board being lit up. However, it is possible if we start with n squares lit up (e.g., start with all the squares on one of the long diagonals lit up). How many configurations of 4 lit up squares will result in the entire 4×4 board being lit up?

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