Quiz 7

Your Name:

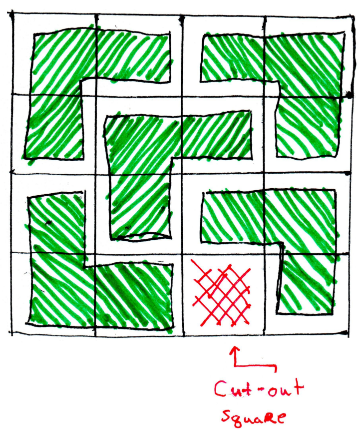
Instructions

This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized*. Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 8 points for a total of 32 points. Good luck and have fun!

Part A

Complete **two** of the following problems.

A1. Consider a grid of squares that is 2^n squares wide by 2^n squares tall such that one of the squares has been cut out, but you don't know which one! You have a bunch of L-shaped trominoes made up of 3 squares. Prove that you can perfectly cover this grid with trominoes (with no overlap) for any $n \in \mathbb{N}$. The figure below depicts one possible covering for the case involving $n = 2$.



A2. The first vote counts of the papal conclave resulted in 33 votes each for candidates A and B and 34 votes for candidate C. The cardinals then discussed the candidates in pairs. In the second round each pair of cardinals with differing first votes changed their votes to the third candidate they did not vote for in the first round. The new vote counts were 16, 17 and 67. They were about to start the smoke signal when Cardinal Ordinal shouted "wait". What was his reason?

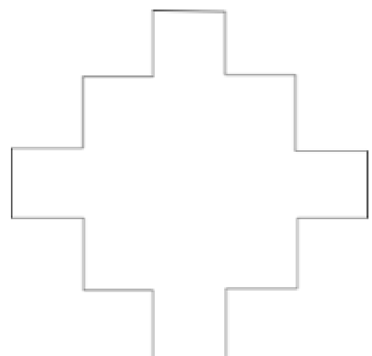
A3. A colony of chameleons on an island currently comprises 13 green, 15 blue, and 17 red individuals. When two chameleons of different colors meet, they both change their colors to the third color. Is it possible that all chameleons in the colony eventually have the same color?

Part B

Complete **two** of the following problems.

B1. A kangaroo jumps along the number line. It starts at a random positive integer s and then starts jumping either to the left or to the right and once it starts jumping in one direction, it keeps jumping in the same direction. If it starts jumping to the right, then every second it jumps n units to the right (the same positive integer n each time). However, if it starts jumping to the left, then every second it jumps $2n$ units to the left. It's dark and we do not know the kangaroo's starting position, we do not know which direction it is hopping, and we do not know what n is. However, at any given second, we are allowed to choose an integer and search there. If the kangaroo is on that integer, we catch it; if not, we have to try again. Devise a strategy for catching the kangaroo.

B2. Cut the following shape into 4 identical pieces that can be re-assembled to form a square.



B3. Consider a gambler who tosses a coin at most 8 times, and if it comes out heads (H), wins a dollar, and if it comes out tails (T), loses a dollar. He is kicked out as soon as he is in the red, i.e., has negative capital. In how many ways can he survive to 8 rounds, but at the end break even?