Quiz 2

Your Name:

Instructions

This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized*. Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 4 points for a total of 16 points. Good luck and have fun!

Part A

Complete **two** of the following problems.

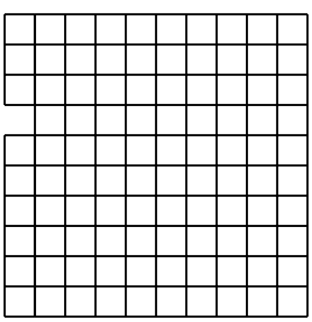
A1. Suppose there are two bags of candy containing 8 pieces and 6 pieces, respectively. You and your friend are going to play a game and the winner gets to eat all of the candy. Here are the rules for the game:

1. You and your friend will alternate removing pieces of candy from the bags. Let's assume that you go first.
2. On each turn, the designated player selects a bag that still has candy in it and then removes at least one piece of candy. The designated player can only remove candy from a single bag and he/she must remove at least one piece.
3. The winner is the one that removes all the candy from the last remaining bag.

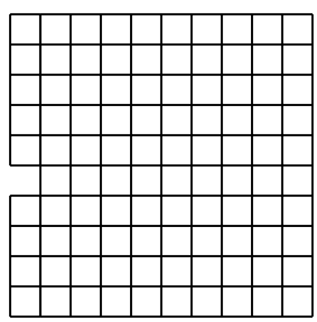
Does one of you have a guaranteed winning strategy? If so, describe that strategy.

A2. Tile each of the grids below with trominoes that consist of 3 squares in a line. If a tiling is not possible, explain way.

(a)



(b)



A3. There is a plate of 40 cookies. You and your friend are going to take turns taking either 1 or 2 cookies from the plate. However, it is a faux pas to take the last cookie, so you want to make sure that you do not take the last cookie. How can you guarantee that you will never be the one taking the last cookie? What about n cookies?

 Written by D.C. Ernst

Part B

Complete **two** of the following problems.

B1. In this problem, we will explore a modified version of the Sylver Coinage Game. In the new version of the game, a fixed positive integer $n \geq 3$ is agreed upon in advance. Then 2 players, A and B, alternately name positive integers from the set $\{1, 2, \ldots, n\}$ that are not the sum of nonnegative multiples of previously named numbers among $\{1, 2, \ldots, n\}$. The person who is forced to name 1 is the loser! Here is a sample game between A and B using the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ (i.e., $n = 10$):

(a) A opens with 4. Now neither player can name $4, 8$.

(b) B names 5. Neither player can name $4, 5, 8, 9, 10$.

(c) A names 6. Neither player can name $4, 5, 6, 8, 9, 10$.

(d) B names 3. Neither player can name $3, 4, 5, 6, 7, 8, 9, 10$.

(e) A names 2. Neither player can name $2, 3, 4, 5, 6, 7, 8, 9, 10$.

(f) B is forced to name 1 and loses.

Suppose player A always goes first. Argue that if there exists an n such that player B is guaranteed to win on the set $\{1, 2, \ldots, n\}$ as long as he or she plays intelligently, then player A is guaranteed to win on the set $\{1, 2, \ldots, n, n+1\}$ as long as he or she plays intelligently. Your argument should describe a strategy for player A.

B2. Two prisoners are locked away in two separate towers, say North Tower and South Tower, and each tower has its own prison guard. Each morning, the respective guards toss a fair coin and then radio the guard in the other tower and report the outcome (heads or tails) of their coin toss. The guard then shows the prisoner in his/her respective tower the outcome of the coin toss in the opposite tower. At this point, each prisoner must guess the outcome of the coin toss that occurred in his/her tower. If at least one of the prisoners guesses correctly, then the prisoners survive another day. If both guess incorrectly, then both will be executed. Is there a strategy that the prisoners can implement that will ensure their survival (until they die of old age in prison) or are they doomed to eventually guess incorrectly and perish? You may assume that prior to being permanently locked up, the prisoners had a few minutes to concoct a plan.

B3. How many factors of 15 are there in 50! (i.e., 50 factorial)?

