

Quiz 4

Your Name:

Instructions

This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized*. Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 4 points for a total of 16 points. Good luck and have fun!

Part A

Complete **two** of the following problems.

- A1. Let X be the intersection of the diagonals of the trapezoid $ABCD$ with parallel sides AB and CD . Show that the areas of triangles AXD and BXC are the same.
- A2. Two different positive numbers a and b each differ from their reciprocal by 1. What is $a + b$?
- A3. Four people are lined up on some steps. They are all looking down the steps and a wall separates the fourth person from the other three. In particular:
- Person 1 can see persons 2 and 3.
 - Person 2 can see person 3.
 - Person 3 cannot see anyone.
 - Person 4 cannot see anyone.

All four people are wearing hats. They are told that there are two white hats and two black hats. Initially, no one knows what color hat they are wearing. They are told to shout out the color of the hat that they are wearing as soon as they know for certain what color it is. Additional constraints:

- They are not allowed to turn around or move.
- They are not allowed to talk to each other.
- They are not allowed to take their hats off.

Who is the first person to shout out the color of his/her hat and why?

Part B

Complete **two** of the following problems.

- B1. In the game Light Up, two players alternately choose unlit squares from an $m \times n$ grid of light-up squares. The objective of the game is to be the first to light up the entire grid. At the beginning of the game, all squares are turned off. On each player's turn, the player selects any square that is currently off and then the selected square gets lit up. Moreover, additional squares get lit up if at least two of its immediate neighbors (horizontal or vertical) are lit up. This process continues until no new squares are lit up and then it is the next player's turn. The loser of the game is the player that no longer has an available square to light up. Determine which player has a winning strategy for a 3×3 grid.

B2. Consider the following dialogue.

William: I have three children.

Harry: What are their ages?

William: The product of their ages is 36.

Harry: I still don't know their ages.

William: The sum of their ages is your apartment number.

Harry: I still don't know their ages!

William: The oldest plays football.

Harry: Now I know their ages.

What are the ages of William's children?

B3. You bought a rectangular puzzle consisting of 253 pieces. Each piece is identical to one of the 5 samples shown in the diagram. Is it possible to re-assemble this puzzle? If so, how many pieces of type E are there in the puzzle? If it's not possible, explain why. You may assume that the puzzle is solvable.
Hint: 253 is divisible by 11.

