

Quiz 7

Your Name:

Instructions

This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized*. Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 4 points for a total of 16 points. Good luck and have fun!

Part A

Complete **two** of the following problems.

- A1. In a PE class, everyone has 5 friends. Friendships are mutual. Two students in the class are appointed captains. The captains take turns selecting members for their teams, until everyone is selected. Prove that at the end of the selection process there are the same number of friendships within each team.
- A2. In the senate of the Klingon home world no senator has more than three enemies. Show that the senate can be separated into two houses so that nobody has more than one enemy in the same house.
- A3. There are $2n$ Federation ambassadors invited to a Ferengi banquet. Every ambassador has at most $n - 1$ enemies. Show that the ambassadors can be seated around a round table avoiding enemies sitting next to each other.

Part B

Complete **two** of the following problems.

- B1. Suppose you have 9 coins, all identical in appearance and weight except for one that **we know is heavier** than the other 8 coins. Is it possible to detect the counterfeit coin in at most two weighings with a two-pan scale? If so, describe an algorithm and explain why it works. If this is impossible, explain why.
- B2. A line of soldiers, standing shoulder to shoulder, is receiving marching commands from a drill instructor. Turn left, says the instructor. Of course there are mistakes. Some of them turn left, and some right. Then the instructor gives the correction command several times. At each command, if two adjacent soldiers face each other, both assume a mistake and turn in the other direction. Does the line become stable after a while? Justify your answer.
- B3. A week before Thanksgiving, a sly turkey is hiding from a family that wants to cook it for the holiday dinner. There are five boxes in a row, and the turkey hides in one of these boxes. Each night, the turkey moves one box to the left or right, hiding in an adjacent box the next day. Each morning, the family can look in one box to try to find the turkey. How can the family guarantee they will find the turkey before Thanksgiving dinner? *Hint*: See if you can first sort out the case when the turkey is in an even-numbered box on the first day.