Quiz 1

Your Name:

Instructions

This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized*. Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 8 points for a total of 32 points. Good luck and have fun!

Part A

Complete **two** of the following problems.

A1. Imagine a hallway with 1000 doors numbered consecutively 1 through 1000. Suppose all of the doors are closed to start with. Then some dude with nothing better to do walks down the hallway and opens all of the doors. Because the dude is still bored, he decides to close every other door starting with door number 2. Then he walks down the hall and changes (i.e., if open, he closes it; if closed, he opens it) every third door starting with door 3. Then he walks down the hall and changes every fourth door starting with door 4. He continues this way, making a total of 1000 passes down the hallway, so that on the 1000th pass, he changes door 1000. At the end of this process, which doors are open and which doors are closed?

A2. Imagine you have 25 pebbles, each occupying one square on a 5 by 5 chess board. Suppose that each pebble must move to an adjacent square by only moving up, down, left, or right. If this is possible, describe a solution. If this is impossible, explain why.

A3. I have 10 sticks in my bag. The length of each stick is an integer. No matter which 3 sticks I try to use, I cannot make a triangle out of those sticks. What is the minimum length of the longest stick?

Part B

Complete **two** of the following problems.

B1. An 8×8 chess board can be tiled with 32 dominos (where each domino covers exactly two adjacent squares of the chess board). Suppose we remove the top left and bottom right corners of the board. Can the remaining 62 squares be tiled by 31 dominos?

B2. Find a visual proof of the following fact. *Warning:* This problem is not about triangular numbers.

For $n \in \mathbb{N}$, $1 + 3 + 5 + \cdots + (2n - 1) = n^2$.

B3. How many ways can 110 be written as the sum of 14 different positive integers? *Hint:* First, figure out what the largest possible integer could be in the sum. Note that the largest integer in the sum will be maximized when the other 13 numbers are as small as possible. One of the formulas we encountered when dealing with triangular numbers could be useful here (but isn't completely necessary). Finish off the problem by doing an analysis of cases.

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