

## Quiz 4

Name:

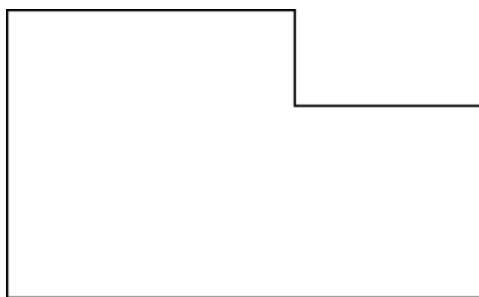
### Instructions

This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized*. Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 8 points for a total of 32 points. Good luck and have fun!

### Part A

Complete **two** of the following problems.

- A1. (The Martian Artifacts) Recent archaeological work on Mars shows a number of sites, each site containing a large mound of white spheres, each about the size of a tennis ball. Each ball contains a jewel. The jewels come in many different colors, but at each site, strictly more than half of the spheres contain jewels of the same color. When two balls are brought together, they both glow white if their internal jewels are the same color; otherwise, no glow. You have recently stumbled on a cache of 13 spheres. In how few tests can you find a sphere that you are certain holds a jewel of the majority color? You must justify your solution.
- A2. (Double Tangent) Find the equation of the line which is tangent to the graph of the function  $f(x) = 72x^4 + 24x^3 - 46x^2 + 19x - 7$  at two points. You must not use calculus and you must justify your solution.
- A3. This shape below is made by joining two squares, one  $3 \times 3$ , one  $2 \times 2$ . Divide it into a few pieces which can be re-assembled to make a square.



### Part B

Complete **two** of the following problems.

- B1. (Deep Roots) If  $b = \sqrt{3 + 2\sqrt{3 + 2\sqrt{3 + 2\sqrt{\dots}}}}$  is a number, what number is it?

B2. (Bin Laden 2) In a giant bin, we stir together 2017 red balls and 2000 (notice the change) green balls. These are our instructions:

1. Pull out three balls at random
2. If they are all three the same color, throw them all away.
3. If they are two of one color and one of the other, stir one of the majority color balls back into the bin, and throw away the other two.

If we follow those instructions repeatedly until there are fewer than three balls left in the bin, there are 6 possibilities for whats left:

1. No balls
2. One red ball
3. One green ball
4. Two red balls
5. One red and one green
6. Two green balls

Are any of these more or less likely than any of the others? Which is most likely? Which is least likely?

B3. (All Different 2) You are to fill each row with the letters *A*, *B*, *C*, *D*. Each row should have all four letters and no two rows are the same. The dots show the only places where vertically adjacent letters are the same.

.	<b>C</b>		
	<b>B</b>	.	<b>A</b>
		<b>C</b>	.
<b>A</b>		.	
		.	<b>C</b>
	<b>B</b>		
.			<b>D</b>
	<b>A</b>		