

Quiz 5

Name:

Instructions

This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized*. Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 8 points for a total of 32 points. Good luck and have fun!

Part A

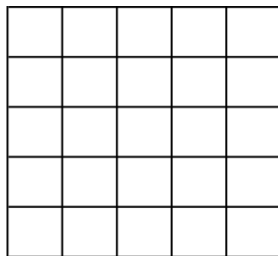
Complete **two** of the following problems.

- A1. Suppose you have $n \geq 2$ coins, all identical in appearance and weight except for one that is either heavier or lighter than the other $n - 1$ coins. Suppose our goal is to identify the counterfeit coin with a two-pan scale using the minimal number of weighings and to be able to determine if the counterfeit coin is heavier or lighter than the remaining coins. Let k denote the number of weighing used to detect the counterfeit coin. For part of Problem 55, we determined that

$$n \leq \frac{3^k - 1}{2}.$$

Determine which numbers of coins we handle in at most (a) 3 weighings and (b) 2 weighings. You may assume that we can handle 12 coins in at most 3 weighings, which we proved previously in class.

- A2. How many ways are there to place the letters A, B, C, D, E into the grid below, one per box, so that each row, each column and each of the two long diagonals contain one of each letter?



- A3. You have 14 coins, dated 1901 through 1914. Seven of these coins are real and weigh 1.000 ounce each. The other seven are counterfeit and weigh 0.999 ounces each. You do not know which coins are real or counterfeit. You also cannot tell which coins are real by look or feel. Fortunately for you, Zoltar the Fortune-Weighing Robot is capable of making very precise measurements. You may place any number of coins in each of Zoltar's two hands and Zoltar will do the following:
- If the weights in each hand are equal, Zoltar tells you so and returns all of the coins.
 - If the weight in one hand is heavier than the weight in the other, then Zoltar takes one coin, at random, from the heavier hand as tribute. Then Zoltar tells you which hand was heavier, and returns the remaining coins to you.

Your objective is to identify a single real coin that Zoltar has not taken as tribute.

Part B

Complete **two** of the following problems.

- B1. A soul swapping machine swaps the souls inside two bodies placed in the machine. Soon after the invention of the machine an unforeseen limitation is discovered: swapping only works on a pair of bodies once. Souls get more and more homesick as they spend time in another body and if a soul is not returned to its original body after a few days, it will kill its current host. Bart (B), Lisa (L), Homer (H), Marge (M), and Ned (N) were involved in a soul-swapping bonanza that resulted in Bart's soul being Lisa's body, Lisa's soul being in Homer's body, Homer's soul being in Marge's body, Marge's soul in Ned's body, and Ned's soul being in Bart's body. Thankfully, Krusty the Clown (K) and Santa's Little Helper (S) never utilized the machine and are available to help put everyone's soul back in the appropriate body. Find a way to return all the souls to their respective owners.
- B2. A certain store sells a product called widgets in boxes of 7, 9, and 11. A number n is called *widgetable* if one can buy exactly n widgets by buying some number of boxes. What is the largest non-widgetable number?
- B3. Cut the following shape into 4 identical pieces which can be re-assembled to form a square.

