Quiz 7

Name:

Instructions

This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized*. Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 8 points for a total of 32 points. Good luck and have fun!

Part A

Complete **two** of the following problems.

A1. Consider an 8×8 grid with light-up squares. In the starting configuration, some subset of the squares are lit up. At each stage, a square lights up if at least two of its immediate neighbors (horizontal or vertical) were "on" during the previous stage. Is it possible for a starting configuration with fewer than 8 squares to cover the entire board? If yes, find it; if no, give a proof.

A2. Annie, Bob, and Cristy are sitting by a campfire when Cristy announces that she is thinking of a 3-digit number. She then tells Annie and Bob that the number she is thinking of is one of the following:

$$515, 516, 519, 617, 618, 714, 716, 814, 815, 817.$$

Next, Cristy whispers the leftmost digit in Annie's ear and then whispers the remaining two digits in Bob's ear. The following conversation then takes place:

> Annie: I don't know what the number is, but I know Bob doesn't know too.
>
> Bob: At first I didn't know what the number was, but now I know.
>
> Annie: Ah, then I know the number, too.

From that information, determine Cristy's number. You must provide sufficient justification.

A3. Suppose we draw n lines in the plane that have the maximum number of unique intersections. This partitions the plane into disjoint regions (some of which are polygons with finite area and some are not). Suppose we color each of the regions so that no two adjacent regions (i.e., share a common edge) have the same color. What is the fewest colors we could use to accomplish this? Justify your answer.

Part B

Complete **two** of the following problems.

B1. A kangaroo jumps along the number line. It starts at a random positive integer s and then starts jumping either to the left or to the right and once it starts jumping in one direction, it keeps jumping in the same direction. If it starts jumping to the right, then every second it jumps n units to the right (the same positive integer n each time). However, if it starts jumping to the left, then every second it jumps $2n$ units to the left. It's dark and we do not know the kangaroo's starting position, we do not know which direction it is hopping, and we do not know what n is. However, at any given second, we are allowed to choose an integer and search there. If the kangaroo is on that integer, we catch it; if not, we have to try again. Devise a strategy for catching the kangaroo.

B2. The inhabitants of planet Nuggetron are obsessed with a game called Nuggetto, which is played by two teams at a time. The ranking of a Nuggetto team is based upon the following rules:

(1) Team X's ranking is the sum of the rankings of the teams that beat Team X.

(2) Team X's ranking is divided evenly by the number of teams that Team X beat.

Suppose Nuggetron has four Nuggetto teams: A, B, C, and D. Suppose that over the course of the season, the following wins occurred:

- Team A beat Team D,
- Team B beat Teams A, C, and D,
- Team C beat Team A,
- Team D beat Team B.

Which team or teams have the highest Nuggetto ranking? You must provide sufficient justification.

B3. A signed permutation of the numbers 1 through n is a fixed arrangement of the numbers 1 through n, where each number can be either be positive or negative. For example, $(-2, 1, -4, 5, 3)$ is a signed permutation of the numbers 1 through 5. In this case, think of positive numbers as being right-side-up and negative numbers as being upside-down. A *reversal* of a signed permutation is the act of performing a 180-degree rotation to some consecutive subsequence of the permutation. That is, a reversal swaps the order of a subsequence of numbers while changing the sign of each number in the subsequence. Performing a reversal to a signed permutation results in a new signed permutation. For example, if we perform a reversal on the second, third, and fourth entries in $(-2, 1, -4, 5, 3)$,we obtain $(-2, -5, 4, -1, 3)$. The *reversal distance* of a signed permutation of 1 through n is the minimum number of reversals required to transform the given signed permutation into $(1, 2, \ldots, n)$. It turns out that the reversal distance of $(3, 1, -2, 4)$ is 3. Find a sequence of 3 reversals that transforms $(3, 1, -2, 4)$ into $(1, 2, 3, 4)$.