Quiz 1

Your Name:

Instructions

This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized*. Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 4 points for a total of 16 points. Good luck and have fun!

Part A

Complete **two** of the following problems.

A1. Imagine a hallway with 1000 doors numbered consecutively 1 through 1000. Suppose all of the doors are closed to start with. Then some dude with nothing better to do walks down the hallway and opens all of the doors. Because the dude is still bored, he decides to close every other door starting with door number 2. Then he walks down the hall and changes (i.e., if open, he closes it; if closed, he opens it) every third door starting with door 3. Then he walks down the hall and changes every fourth door starting with door 4. He continues this way, making a total of 1000 passes down the hallway, so that on the 1000th pass, he changes door 1000. At the end of this process, which doors are open and which doors are closed? You must justify your answer.

A2. I have 10 sticks in my bag. The length of each stick is an integer. No matter which 3 sticks I try to use, I cannot make a triangle out of those sticks. What is the minimum length of the longest stick? You must justify your answer.

A3. Imagine you have 49 pebbles, each occupying one square on a 7 by 7 chess board. Suppose that each pebble must move to an adjacent square by only moving up, down, left, or right. If this is possible, describe a solution. If this is impossible, explain why.

Part B

Complete **two** of the following problems.

B1. Suppose there are two bags of candy containing 8 pieces and 6 pieces, respectively. You and your friend are going to play a game to see who gets all 14 pieces of candy. Here are the rules for the game:

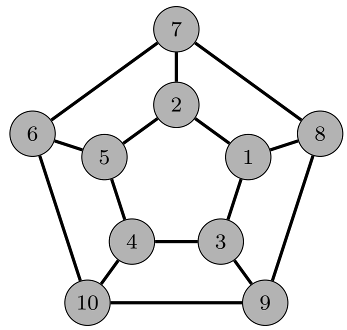
(a) You and your friend will alternate removing pieces of candy from the bags. Let's assume that you go first.

(b) On each turn, the designated player selects a bag that still has candy in it and then removes at least one piece of candy. The designated player can only remove candy from a single bag and he/she must remove at least one piece.

(c) The winner is the one that removes all the candy from the last remaining bag.

Does one of you have a guaranteed winning strategy? If so, describe that strategy.

B2. The graph depicted below is an example of a Hastings Helm. Notice that we have labeled the 10 vertices of the graph with the natural numbers 1 through 10. Two vertices are said to be *adjacent* if they are joined by an edge. For example, the vertex currently labeled by 4 is adjacent to the vertices labeled by 3, 5, and 10. Is it possible to relabel the vertices so that the labels of adjacent vertices have no factors other than 1 in common? Notice that since the vertices currently labeled by 3 and 9 are adjacent and have a factor of 3 in common, the current labeling will not do the job. If you can find an appropriate labeling, then show it. If no such labeling exists, then explain why.



B3. Let a, b, c, d, e, f be distinct elements in the set $\{-3, -2, -1, 2, 4, 6\}$. What is the minimum possible value of $(a+b+c)^2 + (d+e+f)^2$?