Quiz 2

Your Name:

Instructions

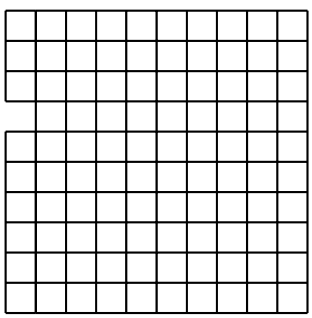
This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized*. Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 4 points for a total of 16 points. Good luck and have fun!

Part A

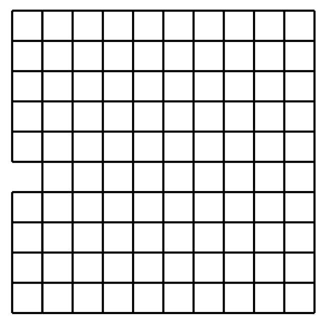
Complete **two** of the following problems.

A1. Tile each of the grids below with trominoes that consist of 3 squares in a line. If a tiling is not possible, explain way.

(a)



(b)



A2. Suppose you randomly cut a stick into 3 pieces. What is the probability that you can form a triangle out of these 3 pieces?

A3. There is a plate of 40 cookies. You and your friend are going to take turns taking either 1 or 2 cookies from the plate. However, it is a faux pas to take the last cookie, so you want to make sure that you do not take the last cookie. How can you guarantee that you will never be the one taking the last cookie? What about n cookies?

Part B

Complete **two** of the following problems.

B1. Let t_n denote the nth triangular number. Find both an algebraic proof and a visual proof of the following fact.

For all $a, b \in \mathbb{N}$, $t_{a+b} = t_a + t_b + ab$.

B2. In this problem, we will explore a modified version of the Sylver Coinage Game. In the new version of the game, a fixed positive integer $n \geq 3$ is agreed upon in advance. Then 2 players, A and B, alternately name positive integers from the set $\{1, 2, \ldots, n\}$ that are not the sum of nonnegative multiples of previously named numbers among $\{1, 2, \ldots, n\}$. The person who is forced to name 1 is the loser! Here is a sample game between A and B using the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ (i.e., $n = 10$):

(a) A opens with 4. Now neither player can name $4, 8$.

(b) B names 5. Neither player can name $4, 5, 8, 9, 10$.

(c) A names 6. Neither player can name $4, 5, 6, 8, 9, 10$.

(d) B names 3. Neither player can name $3, 4, 5, 6, 7, 8, 9, 10$.

(e) A names 2. Neither player can name $2, 3, 4, 5, 6, 7, 8, 9, 10$.

(f) B is forced to name 1 and loses.

Suppose player A always goes first. Argue that if there exists an n such that player B is guaranteed to win on the set $\{1, 2, \ldots, n\}$ as long as he or she plays intelligently, then player A is guaranteed to win on the set $\{1, 2, \ldots, n, n+1\}$ as long as he or she plays intelligently. Your argument should describe a strategy for player A.

B3. Let a, b, c, d, e, f, g, h be distinct elements in the set $\{-7, -5, -3, -2, 2, 4, 6, 13\}$. What is the minimum possible value of $(a+b+c+d)^2 + (e+f+g+h)^2$?