Quiz 4

Your Name:

Instructions

This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized*. Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 4 points for a total of 16 points. Good luck and have fun!

Part A

Complete **two** of the following problems.

A1. We have two strings of pyrotechnic fuse. The strings do not look homogeneous in thickness but both of them have a label saying 4 minutes. So we can assume that it takes 4 minutes to burn through either of these fuses. How can we measure a one minute interval?

A2. In the game Light Up, two players alternately choose unlit squares from an $m \times n$ grid of light-up squares. The objective of the game is to be the first to light up the entire grid. At the beginning of the game, all squares are turned off. On each player's turn, the player selects any square that is currently off and then the selected square gets lit up. Moreover, additional squares get lit up if at least two of its immediate neighbors (horizontal or vertical) are lit up. This process continues until no new squares are lit up and then it is the next player's turn. The loser of the game is the player that no longer has an available square to light up. Determine which player has a winning strategy for a 3×3 grid.

A3. My Uncle Robert owns a stable with 25 race horses. He wants to know which three are the fastest. He owns a race track that can accommodate five horses at a time. What is the minimum number of races required to determine the fastest three horses? *Note:* As long as your answer is correct, you do not need to argue that your solution is optimal.

Part B

Complete **two** of the following problems.

B1. A signed permutation of the numbers 1 through n is a fixed arrangement of the numbers 1 through n, where each number can be either be positive or negative. For example, $(-2, 1, -4, 5, 3)$ is a signed permutation of the numbers 1 through 5. A *reversal* of a signed permutation swaps the order of a consecutive subsequence of numbers while changing the sign of each number in the subsequence. Performing a reversal to a signed permutation results in a new signed permutation. For example, if we perform a reversal on the second, third, and fourth entries in $(-2, 1, -4, 5, 3)$, we obtain $(-2, -5, 4, -1, 3)$. The *reversal distance* of a signed permutation of 1 through n is the minimum number of reversals required to transform the given signed permutation into $(1, 2, \ldots, n)$. It turns out that the reversal distance of $(3, 1, -2, 4)$ is 3. Find a sequence of 3 reversals that transforms $(3, 1, -2, 4)$ into $(1, 2, 3, 4)$.

B2. If $b = \sqrt{3 + 2\sqrt{3 + 2\sqrt{3 + 2\sqrt{\cdots}}}}$ is a number, what number is it?

 Written by D.C. Ernst

B3. You are to fill each row with the letters A, B, C, D. Each row should have all four letters and no two rows are the same. The dots show the only places where vertically adjacent letters are the same.