

Quiz 6

Your Name:

Instructions

This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized*. Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 4 points for a total of 16 points. Good luck and have fun!

Part A

Complete **two** of the following problems.

- A1. Show that in any set of seven different positive integers there are three numbers such that the greatest common divisor of any two of them leaves the same remainder when divided by three.
- A2. In the senate of the Klingon home world no senator has more than three enemies. Show that the senate can be separated into two houses so that nobody has more than one enemy in the same house.
- A3. 100 prisoners are isolated in individual jail cells with no way to communicate. They are currently serving life sentences. Due to an overcrowded prison, the jailer decides to offer the prisoners the following deal. There is a room with nothing in it except a light switch (that starts in the off position). At random, the jailer will escort a single prisoner into the room with the light switch. After 5 seconds, the jailer will escort the prisoner back to his/her jail cell. The jailer will repeat this over and over again. He tells each of the prisoners that if one of the prisoners can indicate when every prisoner has been in the room with the light switch at least once, he will let all the prisoners go. However, if a prisoner erroneously states that each prisoner has been in the room with the light switch, then all the prisoners will be executed. Before beginning, the jailer gets all 100 prisoners together and gives them 5 minutes to come up with a plan. What should their plan be? It's important to note that the jailer is choosing prisoners at random to take in the room. That is, by chance, the same prisoner may be escorted to the room several times in a row. Also, your task is to devise a scheme for the prisoners to communicate with the light switch. You shouldn't bother searching for other ways for the prisoners to communicate.

Part B

Complete **two** of the following problems.

- B1. Recent archaeological work on Mars discovered a site containing a pile of white spheres, each about the size of a tennis ball. A plaque near the mound states that each sphere contains a jewel that come in many different colors while strictly more than half of the spheres contain jewels of the same color. When two spheres are brought together, they both glow white if their internal jewels are the same color; otherwise, no glow. In how few tests can you find a sphere that you are certain holds a jewel of the majority color if the number of spheres in the pile is 7? You should provide sufficient justification.

B2. The inhabitants of a certain planet use not four but five basic arithmetic operations. The operations of addition, multiplication, subtraction and division are the same as ours, but they also have a special operation denoted by the sign $@$. We do not know exactly how this operation works, but we have found out that the following properties are valid for all x and y :

(a) $x@0 = x$

(b) $x@y = y@x$

(c) $(x + 1)@y = (x@y) + y + 1$

What is the value of $12@5$ on this planet?

B3. Find positive odd integers $A < B < C$ such that

$$\frac{1}{11} = \frac{1}{A} + \frac{1}{B} + \frac{1}{C}.$$