

Quiz 1

Your Name:

Instructions

This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized*. Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 4 points for a total of 16 points. Good luck and have fun!

Part A

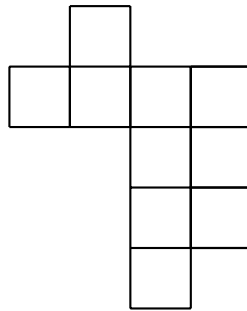
Complete **two** of the following problems.

- A1. Imagine a hallway with 1000 doors numbered consecutively 1 through 1000. Suppose all of the doors are closed to start with. Then some dude with nothing better to do walks down the hallway and opens all of the doors. Because the dude is still bored, he decides to close every other door starting with door number 2. Then he walks down the hall and changes (i.e., if open, he closes it; if closed, he opens it) every third door starting with door 3. Then he walks down the hall and changes every fourth door starting with door 4. He continues this way, making a total of 1000 passes down the hallway, so that on the 1000th pass, he changes door 1000. At the end of this process, which doors are open and which doors are closed? You must justify your answer.
- A2. The Sunny Day Juice Stand sells freshly squeezed lemonade and orange juice at the farmers' market. The juices are ladled out of large glass jars, each holding exactly the same amount of juice. Linda and Julie set up their stand early one Saturday morning. The first customer of the day ordered orange juice and Linda carefully ladled out 8 ounces into a paper cup. As she was about to hand the cup to the customer, he changed his mind and asked for lemonade instead. Accidentally, Linda dumped the cup of orange juice into the jar of lemonade. She quickly mixed up the juices, ladled out a cup of the mixture (mostly lemonade) and turned to hand it to the customer. "I've decided I don't want anything to drink right now," he said, and frazzled, Linda dumped the cupful of juice mixture into the orange juice jar. Linda's assistant, Julie, watched all of this with amusement. As the man walked away, she wondered aloud, "Now is there more orange juice in the lemonade or more lemonade in the orange juice?"
- A3. A soul swapping machine swaps the souls inside two bodies placed in the machine. Soon after the invention of the machine an unforeseen limitation is discovered: swapping only works on a pair of bodies once. Souls get more and more homesick as they spend time in another body and if a soul is not returned to its original body after a few days, it will kill its current host. Suppose the soul swapping machine is used by the following pair of bodies (in the order listed): Adam and Alicia, Alicia and Gwen, Gwen and Blake. In addition, Pharrell and Miley are standing nearby. Is it possible to return the swapped souls back to their original bodies? If so, find a solution that minimizes the number of times the soul swapping machine must be used.

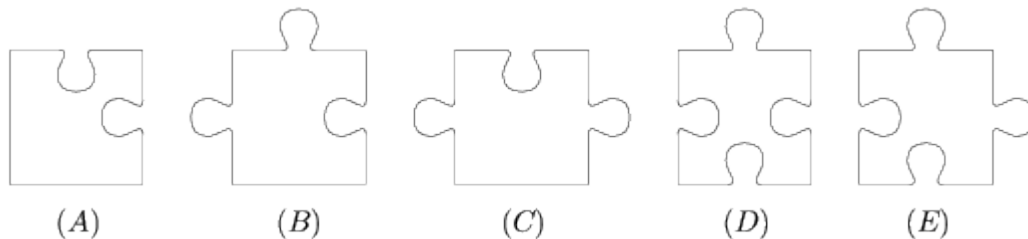
Part B

Complete **two** of the following problems.

- B1. Imagine you have 10 pebbles, each occupying one square of the shape below. Determine whether each of the following scenarios is possible. If so, describe a solution. If a scenario is impossible, explain why. If you choose to do this problem, you need to complete both parts.
- Suppose that each pebble simultaneously must move to an adjacent square by only moving up, down, left, or right. Two pebbles may swap positions, but this is not necessary.
 - Now suppose that all but one pebble (your choice which one) must move to an adjacent square by only moving up, down, left, or right.



- B2. You bought a rectangular puzzle consisting of 253 pieces. Each piece is identical to one of the 5 samples shown in the diagram. Is it possible to re-assemble this puzzle? If so, how many pieces of type E are there in the puzzle? If it's not possible, explain why. You may assume that the puzzle is solvable.
Hint: 253 is divisible by 11.



- B3. Two different positive numbers a and b each differ from their reciprocal by 1. What is $a + b$?