Quiz 4

Your Name:

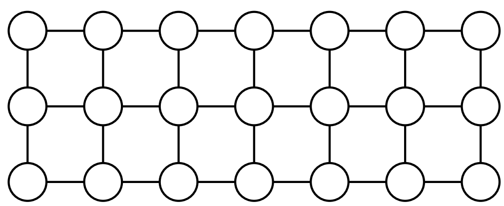
Instructions

This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized*. Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 4 points for a total of 16 points. Good luck and have fun!

Part A

Complete **two** of the following problems.

A1. In the lattice below, we color 11 vertices points black. Prove that no matter which 11 are colored black, we always have a rectangle with black corners.



A2. Consider our Star Base from Problem 44. Recall that our hyper drive allows us to jump from coordinates (a, b) to either coordinates $(a, a + b)$ or to coordinates $(a + b, b)$. If we start at $(1, 0)$, which points in the plane can we get to by using our hyper drive? Justify your answer.

A3. Suppose you randomly cut a stick into 3 pieces. What is the probability that you can form a triangle out of these 3 pieces?

Part B

Complete **two** of the following problems.

B1. A signed permutation of the numbers 1 through n is a fixed arrangement of the numbers 1 through n, where each number can be either be positive or negative. For example, $(-2, 1, -4, 5, 3)$ is a signed permutation of the numbers 1 through 5. In this case, think of positive numbers as being right-side-up and negative numbers as being upside-down. A *reversal* of a signed permutation is the act of performing a 180-degree rotation to some consecutive subsequence of the permutation. That is, a reversal swaps the order of a subsequence of numbers while changing the sign of each number in the subsequence. Performing a reversal to a signed permutation results in a new signed permutation. For example, if we perform a reversal on the second, third, and fourth entries in $(-2, 1, -4, 5, 3)$, we obtain $(-2, -5, 4, -1, 3)$. The *reversal distance* of a signed permutation of 1 through n is the minimum number of reversals required to transform the given signed permutation into $(1, 2, \ldots, n)$. It turns out that the reversal distance of $(3, 1, -2, 4)$ is 3. Find a sequence of 3 reversals that transforms $(3, 1, -2, 4)$ into $(1, 2, 3, 4)$.

B2. Alice and Brenda both ran in a 100-meter race. When Alice crossed the finish line, Brenda was 10 meters behind her. Assuming the girls run the same rate, how many meters behind Brenda should Alice start in order for them to finish in a tie?

B3. Determine whether we can obtain every sequence of length 3 in the Circle-Dot system. You do not need to reproduce the proofs for sequences we've already obtained, but you should cite the appropriate theorem or proof.

Circle-Dot

Circle-Dot begins with two words; called axioms. Using the two axioms and three rules of inference, we can create new Circle-Dot words, which are theorems in the Circle-Dot System. The process of creating Circle-Dot words using the axioms and rules of inference are proofs in the system. Below are the axioms for Circle-Dot. Note that $\circ$ and $\bullet$ are valid symbols in the system while w and v are variables that stand for any nonempty sequence of $\circ$'s and $\bullet$'s.

Axiom A. $\circ\bullet$

Axiom B. $\bullet\circ$

At any time in your proof, you may quote an axiom. Below are the rules for generating new statements from known statements.

Rule 1. Given wv and vw, conclude w

Rule 2. Given w and v, conclude $w \bullet v$

Rule 3. Given $wv\bullet$, conclude $w\circ$

In class, we proved the following theorems.

Theorem C. $\bullet$

Theorem D. $\circ$

Theorem E. $\bullet \bullet \bullet$

Theorem F. $\bullet \bullet \circ$

Theorem G. $\bullet \circ \circ$

Theorem H. $\circ \bullet \bullet\circ$

Theorem I. $\circ \circ \circ\circ$

Theorem J. $\bullet \circ \bullet$

Theorem K. $\bullet \circ \circ\circ$

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