Exam 3 (Part 2)

Your Name:

Names of Any Collaborators:

Instructions

Submit your solutions to the following questions by the beginning of class on **Friday, April 25**. This part of Exam 3 is worth a total of 16 points and is worth 50% of your overall score on Exam 3. Your overall score on Exam 3 is worth 15% of your overall grade.

I expect your solutions to be *well-written, neat, and organized*. Do not turn in rough drafts. What you turn in should be the "polished" version of potentially several drafts. Feel free to type up your final version. The LaTeX source file of this exam is also available if you are interested in typing up your solutions using LaTeX. I'll gladly help you do this if you'd like.

Reviewing material from previous courses and looking up definitions and theorems you may have forgotten is fair game. However, when it comes to completing the following problems, you should *not* look to resources outside the context of this course for help. That is, you should not be consulting the web, other texts, other faculty, or students outside of our course in an attempt to find solutions to the problems you are assigned. This includes ChatGPT, Chegg, and Course Hero. On the other hand, you may use each other, the textbook, me, and your own intuition. Further information:

1. You may freely use any theorems that we have discussed in class, but you should make it clear where you are using a previous result and which result you are using. For example, if a sentence in your proof follows from Theorem X.Y, then you should say so.
2. Unless you prove them, you cannot use any results from the course notes that we have not yet covered.
3. You are **NOT** allowed to consult external sources when working on the exam. This includes people outside of the class, other textbooks, and online resources.
4. You are **NOT** allowed to copy someone else's work.
5. You are **NOT** allowed to let someone else copy your work.
6. You are allowed to discuss the problems with each other and critique each other's work.

I will vigorously pursue anyone suspected of breaking these rules.

You should **turn in this cover page** and all of the work that you have decided to submit. **Please write your solutions and proofs on your own paper.** To convince me that you have read and understand the instructions, sign in the box below.

Signature:

Good luck and have fun!

You may need to digest some new content in the book to complete the following problems. We will spend some time discussing these concepts in class during the time you have available to work on the exam. However, you should not wait until I have discussed the relevant topics. Just dig in and get started.

When completing each of the tasks below, you may utilize any result in the book that comes before the particular problem/theorem, regardless of whether you proved the previous result or not.

1. (4 points each) Complete **two** of the following.
   (a) (Theorem 8.44) Let $f : X \to Y$ be a function and define $\sim$ on X via $a \sim b$ if $f(a) = f(b)$. Prove that $\sim$ is an equivalence relation on X.
   (b) (Theorem 8.60) Prove that if $f : X \to Y$ and $g : Y \to Z$ are both surjective functions, then $g \circ f$ is also surjective.
   (c) (Theorem 8.61) Prove that if $f : X \to Y$ and $g : Y \to Z$ are both injective functions, then $g \circ f$ is also injective.
2. (Theorem 8.46, 4 points) Let $f : X \to Y$ be a function and define $\sim$ on X as in Theorem 8.44. Prove that the function $\overline{f} : X/\sim \to \text{Rng}(f)$ defined via $\overline{f}([a]) = f(a)$ is a bijection. *Note:* When proving this theorem, the first thing you should do is verify that the description for $\overline{f}$ is well defined.
3. (2 points each) For each of the following, identify the equivalence classes induced by the relation from Theorem 8.44 for the given function.
   (a) The function f defined in Example 8.2.
   (b) The function c defined in Problem 8.35(e). Can you describe the equivalence classes geometrically?

