

Homework 10

Abstract Algebra I

Complete the following problems. Note that you should only use results that we've discussed so far this semester.

Problem 1. Prove that if G is an abelian group of order pq , where p and q are distinct primes, then G is cyclic. *Hint:* Use Problem 1 from Homework 8.

Problem 2. Prove that $|\text{Aut}(D_8)| \leq 8$ by first proving that under any automorphism, r has at most two possible images and s has at most four possible images.

Problem 3. Prove that characteristic subgroups are normal.

Problem 4. Provide an example of a normal subgroup that is not characteristic.

Problem 5. Prove that if H is the unique subgroup of a given order in G , then H is characteristic. *Hint:* This problem is as easy as quoting a previous homework problem.

Problem 6. Prove **one** of the following.

- (a) If H is characteristic in K and K is normal in G , then H is normal in G .
- (b) If H is characteristic in K and K is characteristic in G , then H is characteristic in G .