

Homework 9

Abstract Algebra II

Complete the following problems. Note that you should only use results that we've discussed so far this semester or last semester.

Problem 1. Determine the Galois group of $f(x) = (x^2 - 2)(x^2 - 3)(x^2 - 5)$ over \mathbb{Q} .

Problem 2. Determine the Galois group of the splitting field over \mathbb{Q} of $g(x) = x^4 - 14x^2 + 9$.

Problem 3. Let $K = \mathbb{Q}(\sqrt[8]{2}, i)$, $F_1 = \mathbb{Q}(i)$, and $F_2 = \mathbb{Q}(\sqrt{2})$.

(a) Prove that $\text{Gal}(K/F_1) \cong \mathbb{Z}_8$.

(b) Prove that $\text{Gal}(K/F_2) \cong D_8$.

Problem 4. Let $f(x) \in \mathbb{Q}[x]$. Suppose that $z \in \mathbb{C}$ is a root of $f(x)$.

(a) Prove that \bar{z} (complex conjugate of z) is also a root of $f(x)$.

(b) Suppose $f(x)$ has degree 3. Prove that if the Galois group of the splitting field of $f(x)$ is isomorphic to \mathbb{Z}_3 , then $f(x)$ has only real roots.