Final Exam

Your Name:

Names of Any Collaborators:

Instructions

Answer each of the following questions by 9:00AM on **Friday, May 10**. This exam is worth a total of 25 points and is worth 25% of your overall grade.

I expect your solutions to be *well-written, neat, and organized*. Do not turn in rough drafts. What you turn in should be the "polished" version of potentially several drafts. Feel free to type up your final version. The LaTeX source file of this exam is also available if you are interested in typing up your solutions using LaTeX. I'll gladly help you do this if you'd like.

Reviewing material from previous courses and looking up definitions and theorems you may have forgotten is fair game. However, when it comes to completing the following problems, you should *not* look to resources outside the context of this course for help. That is, you should not be consulting the web, other texts, other faculty, or students outside of our course in an attempt to find solutions to the problems you are assigned. This includes ChatGPT, Chegg, and Course Hero. On the other hand, you may use each other, class notes, me, and your own intuition. Further information:

1. You may freely use any theorems that we have discussed in class, but you should make it clear where you are using a previous result and which result you are using.
2. Unless you prove them, you cannot use any results from the course notes that we have not yet covered.
3. You are **NOT** allowed to consult external sources when working on the exam. This includes people outside of the class, other textbooks, and online resources.
4. You are **NOT** allowed to copy someone else's work.
5. You are **NOT** allowed to let someone else copy your work.
6. You are allowed to discuss the problems with each other and critique each other's work.

I will vigorously pursue anyone suspected of breaking these rules.

You should **turn in this cover page** and all of the work that you have decided to submit. **Please write your solutions and proofs on your own paper.** To convince me that you have read and understand the instructions, sign in the box below.

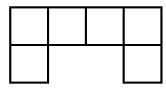
Signature:

Good luck and have fun!

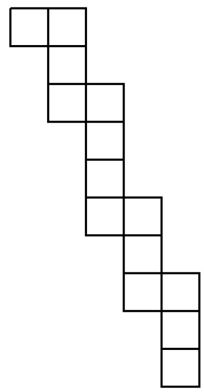
Important: Most of the problems below ask you to prove theorems introduced in class. In each of those cases, you can only rely on definitions and results that came before the corresponding proposition. Also, you are welcome to use any earlier result on this exam to prove a later result, even if you did not prove the earlier result.

1. (3 points each) Complete **three** of the following.

   (a) Draw a game tree for the following Domineering position. You may identify symmetric positions. Label each position with the corresponding outcome class (i.e., $\mathcal{L}$, $\mathcal{R}$, $\mathcal{P}$, $\mathcal{N}$).

   (b) Determine the outcome class of the following Domineering position.

   (c) The partizan subtraction game $\textsc{Subtraction}_n(1,2 \mid 2,3)$ is played with a single heap of n tokens. A move by L is to remove one or two tokens while a move by R is to remove two or three tokens. Determine the outcome class for $\textsc{Subtraction}_n(1,2 \mid 2,3)$ for all $n \in \mathbb{N} \cup \{0\}$. Your answer will look like a piecewise function and should use "mod". Prove that your answer is correct.

   (d) Prove that $\{\{4 \mid 1\}, 0 \mid 2\} = 1$.

   (e) Which sums (if any) of two games (not necessarily distinct) among 0, 1, -1, and $*$ are born by day 1? Justify your answer.

2. (4 points each) Complete **four** of the following. All of these are propositions stated in class.

   (a) Let G be any game and let $Z \in \mathcal{P}$. Prove that $o(G) = o(G + Z)$.

   (b) Let G be any game. Prove that $G + 0 = G$.

   (c) Let G, H, and K be games. Prove that $(G + H) + K = G + (H + K)$.

   (d) Let G and H be games. Prove that $-(G + H) = -G + -H$.

   (e) Let G be a game. Prove that $G = 0$ if and only if G is a $\mathcal{P}$-position.

   (f) Let G and H be games. Prove that $G = H$ if and only if $G + X = H + X$ for all games X.

   (g) Let G be a game. Prove that if L wins moving second on G, then for all games X, $o_L(G + X) \geq o_L(X)$.[*]

   (h) Prove that the relation $\geq$ is a partial order on the class of games.

[*]This is (2) implies (4) of the proposition involving four parts that are all equivalent.