Math 1300: Calculus I, Fall 2006; Instructor: Dana Ernst
Section 5.3: More on Curve Sketching (Part I)

We are interested in the following features of the graph of a function:

- symmetries
- x-intercepts
- y-intercepts
- vertical asymptotes
- behavior as $x \to \infty$ and $x \to -\infty$ (horizontal asymptote)
- intervals of increase, intervals of decrease
- relative extrema
- intervals of concavity
- inflection points

In general, we will be given a function and asked to sketch its graph. To do this, we will have to identify some or all of the features above. Let's first try to sketch the graph of a function where all of the above information is given.

Example 1: Sketch the graph of the function that has the following properties.

1. $f(-5) = 0, f(-3) = -3, f(-2) = 0$
2. $f(-1.5) = .5, f(-.5) = 1, f(1.5) = 2.5$
3. $\lim\limits_{x \to 0} f(x) = \infty$ and $\lim\limits_{x \to 3} f(x) = \infty$
4. $\lim_{x \to \infty} f(x) = 1$ and $\lim_{x \to -\infty} f(x) = 1$
5. $f'(-3)$ undefined
6. $f'(1.5) = 0, f'(-1.5) = 0$
7. $f'(x) > 0$ on $(-3, 0)$ and $(1.5, 3)$
8. $f'(x) < 0$ on $(-\infty, -3), (0, 1.5)$, and $(3, \infty)$
9. $f''(x) > 0$ on $(-1.5, 0), (0, 3)$, and $(3, \infty)$
10. $f''(x) < 0$ on $(-\infty, -3)$ and $(-3, -1.5)$

Guidelines for Sketching Graphs of Functions

1. Determine whether there is symmetry about the y-axis or the origin.
2. Find x and y-intercepts.
3. Identify vertical asymptotes.
4. Determine end behavior by computing limits of $f(x)$ as $x \to \infty$ and $x \to -\infty$ (Does graph have any horizontal asymptotes?).
5. Find critical points, determine intervals of increase and decrease, and identify any relative extrema.
6. Find x-values where $f''(x) = 0$ or is undefined, determine intervals of concavity, and identify any inflection points

Let's start by sketching the graphs of some rational functions.

Example 2: Sketch the graph of the following functions. (You probably don't have room to do these examples on here, so use another sheet of paper.)

(a) $f(x) = \dfrac{2(x^2-9)}{x^2-4}$

(b) $g(x) = \dfrac{-x}{(x^2-1)^2}$