MA 25560: Calculus II (Fall 2009)
Review of Calculus I

NAME:

Instructions: In order to review some of the main topics from Calculus I, I want to you answer the following questions. For those of you that had me last semester in Calculus I, you'll recognize these questions from our final exam. The purpose of this assignment is to get your brain thinking about calculus again. Don't panic if there are a couple questions that you don't remember how to do. However, if you find yourself struggling significantly, then we should talk. I'd like you to turn this in on **Friday, 9.4**. Please work together on this!

1. Consider the following function.

$$f(x) = \begin{cases} x^2 + 1, & x < 1 \\ x - 4, & x \geq 1 \end{cases}$$

For (a)–(e), evaluate the expression. If a limit does not exist, specify whether the limit equals ∞, $-\infty$, or simply does not exist (in which case, write DNE). For (a)–(d), you do *not* need to justify your answer.

(a) $\lim_{x \to 1^-} f(x) =$

(b) $\lim_{x \to 1^+} f(x) =$

(c) $\lim_{x \to 1} f(x) =$

(d) $f(1) =$

(e) Is f continuous at $x = 1$? Justify your answer.

2. Evaluate each of the following limits. If a limit does not exist, specify whether the limit equals ∞, $-\infty$, or simply does not exist (in which case, write DNE). Sufficient work must be shown. Give *exact answers*.

   (a) $\displaystyle\lim_{x\to 2} \frac{x-2}{x^2-4}$

   (b) $\displaystyle\lim_{h\to 0} \frac{(x+h)^2+1-(x^2+1)}{h}$ (Hint: your answer should be a function of x.)

   (c) $\displaystyle\lim_{x\to \infty} \frac{3x^2-5x+1}{4-x^2}$

3. Differentiate each of the following functions. You do *not* need to simplify your answers, but sufficient work must be shown to receive full credit.

   (a) $f(x) = \dfrac{3x^4}{2} + x^2\sqrt{3} - \dfrac{3}{x^2} + \sqrt{2}$

   (b) $y = \sqrt{\sin 3x}$

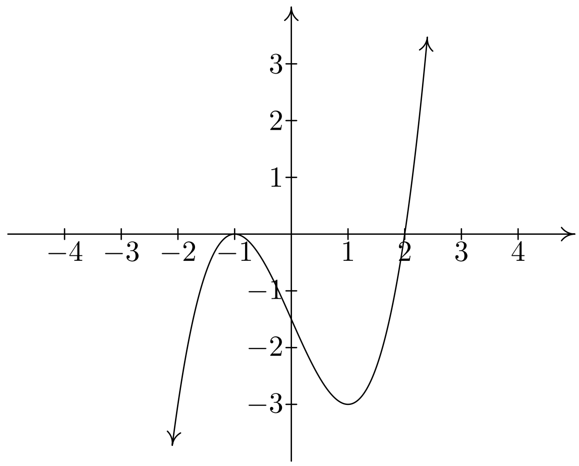
   (c) $g(x) = \dfrac{x}{1 - x^2}$

   (d) $A(x) = \displaystyle\int_0^x \cos t \; dt$

4. Find $\dfrac{dy}{dx}$ if $x^3 + y^3 = xy$. You do *not* need to simplify your answer, but you should solve for $\dfrac{dy}{dx}$.

5. Let $f(x) = \cos x$, find the *equation* of the tangent line to the graph of f when $x = \pi/6$. It does not matter what form the equation of the line takes, but all coefficients should have exact values (i.e., no decimal approximations).

6. Let f be a differentiable function. Suppose that the following graph is the graph of the *derivative* of f (i.e., the graph of f').



Graph of f'

(a) Find the x-coordinates of all points on the graph of $f(x)$ where the tangent line is horizontal.

(b) Find the intervals, if any, on which $f(x)$ is increasing.

(c) Find the intervals, if any, on which $f(x)$ is decreasing.

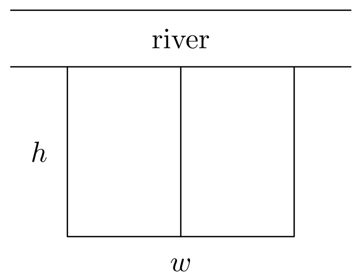
(d) Find the x-coordinates, if any, where $f(x)$ attains a local max.

(e) Find the x-coordinates, if any, where $f(x)$ attains a local min.

(f) Find the x-coordinates, if any, of all of the inflection points of $f(x)$.

7. A large spherical meteor-nugget is speeding towards Earth. If the radius of the meteorite is decreasing at a rate of 1/4 mile per day, what is the rate of change in the volume of the meteorite when the radius is 5 miles? Give an *exact* answer. Your answer should be labeled with appropriate units. (Hint: the volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.)

8. A farmer has 1200 feet of fencing with which to enclose a pasture for grazing nuggets. The farmer only needs to enclose 3 sides of the pasture since the remaining side is bounded by a river (no, nuggets can't swim). In addition, some of the nuggets don't get along with some of the other nuggets. He plans to separate the troublesome nuggets by forming two adjacent corrals (see figure).



   (a) Let A represent the area of the rectangular pen. Find an equation for A that involves only the variable h.

   (b) Find the feasible domain for A. (Hint: how small can h be? How large can h be?)

   (c) Using your answers to (a) and (b), determine the *dimensions* that will maximize the area of the rectangular pen. (Justifying your answer will not only make sure that you receive full credit, but will also ensure that you don't make a mistake.)

9. Evaluate each of the following integrals. Sufficient work must be shown.

   (a) $\displaystyle\int \frac{4 - x^3}{x^2}\, dx$

   (b) $\displaystyle\int_0^1 x^2\sqrt{1 - x^3}\, dx$

(c) $\displaystyle\int \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$

10. Find the area of the region bounded by the graphs of $f(x) = 4x$ and $g(x) = x^2 - 5$.

11. Setup (but do *not* evaluate) an integral that would determine the volume of the solid obtained by revolving the region bounded by the graphs of $y = 2 - x^2$ and $y = x^2$ about the x-axis.

12. Setup (but do *not* evaluate) an integral that would determine the volume of the solid obtained by revolving the region bounded by the graphs of $f(x) = 1$ and $g(x) = x^2$ about the line $x = 2$.

13. Provide an example of each of the following. You do *not* need to justify your answer.

    (a) An *equation* of a function f that is continuous everywhere, but not differentiable at $x = 0$.

    (b) An *equation* of a function g such that g has a critical number at $x = 0$, but g does not have a local maximum or local minimum at $x = 0$.

    (c) An *equation* of a function h such that h has a local maximum at $x = 0$, but $h'(0) \neq 0$.

    (d) An *equation* of a function k such that $k''(0) = 0$, but k does *not* have an inflection point at $x = 0$.

14. A common theme in Calculus I is to start with an approximation for something that is seemingly difficult to compute and then take a limit to get an exact answer. Describe ONE such situation that arises in Calculus I. I'm looking for an intuitive understanding, but you should provide some detail using proper notation. (Using pictures to aid in your description will be very useful.)

15. **Bonus Question 1:** What is it that we computed in problem 2b? Be as specific as possible.

16. **Bonus Question 2:** Describe the function $A(x)$ given in problem 3d? Drawing the appropriate picture will convince me you understand what this function is. (Note: I want you to describe $A(x)$, not $A'(x)$.)