Lab 1: Review of current integration techniques

Names:

Goal

The goal of this lab is to review some of our current integration techniques and to introduce a few new (and more useful) integration formulas.

Directions

In groups of 2–4, answer each of the following questions in the space provided. You only need to turn in one lab per group (make sure you put everyone's name on this sheet). The lab is due on **Monday, September 28** and is worth 10 points.

Exercises

1. First, let's collect some of our recent formulas, so that we have them all in one place. By looking through your notes or the book, complete the following table.

$\int \frac{1}{u}\, du =$	$\int e^u\, du =$
$\int \frac{1}{\sqrt{1-u^2}}\, du =$	$\int \frac{1}{u^2+1}\, du =$
$\int \frac{1}{u\sqrt{u^2-1}}\, du =$	$\int \frac{1}{\sqrt{u^2+1}}\, du =$
$\int \frac{1}{\sqrt{u^2-1}}\, du =$	$\int \frac{1}{1-u^2}\, du =$

   We have encountered more formulas than the ones above, but these are a subset of the really important ones. You should memorize these.

   The last six of these will really be the focus of this lab, but the first two, along with ordinary u-substitution will come up, as well.

   Notice that the last six formulas all look pretty similar. One thing worth mentioning is that u^2 occurs in each of the last six formulas and it is either being added to 1, subtracted from 1, or 1 is being subtracted from u^2 (order of subtraction matters!).

2. One thing you should have noticed when doing your homework is that many functions have something else in place of the 1 that occurs in the last six formulas above. By now, you should be aware of how much of a nuisance this can be.

To illustrate the point (and to create a new and much more useful formula), integrate the following, where a is some real number. (You need to do some factoring and rewriting before you can use one of the formulas.)

$$\int \frac{1}{\sqrt{a^2 - x^2}}\, dx =$$

From now on, we can just use the formula we produced above when we have some constant a that is being squared instead of a 1.

3. Using the formula from the previous problem, integrate the following function.

$$\int \frac{1}{\sqrt{9 - x^2}}\, dx$$

4. Now, if we were extremely motivated, we could do the same thing for each of the remaining formulas. Thankfully, we don't need to do that. Flip to page 6 of the Reference Pages (located at the back of your book). Notice that we derived formula 16 in the previous exercise. Using the table of integrals on page 6 of the Reference Pages, complete the following table. (Be very careful; many of them look nearly identical.)

$\int \frac{1}{\sqrt{a^2 - u^2}}\, du =$	$\int \frac{1}{u^2 + a^2}\, du =$
$\int \frac{1}{u\sqrt{u^2 - a^2}}\, du =$	$\int \frac{1}{\sqrt{u^2 + a^2}}\, du =$
$\int \frac{1}{\sqrt{u^2 - a^2}}\, du =$	$\int \frac{1}{a^2 - u^2}\, du =$

From now on, you can use these fancier formulas instead of doing all of the initial factoring and rewriting.

5. Now, wait a second! The formulas that you were supposed to write down for the last three in the table (assuming we are counting left to right, then down) don't look anything like the formulas you wrote down in exercise 1. What's up with that? Reconcile the differences and explain below. (Hint: look at page 4 of the notes from 7.7 or look at page 466 in the book.)

   To convince me that you really understand, write equivalent formulas for the following integrals that look more like the ones in the box on the first page.

   $$\int \frac{1}{\sqrt{u^2+a^2}}\,du = \qquad \int \frac{1}{\sqrt{u^2-a^2}}\,du =$$

   $$\int \frac{1}{a^2-u^2}\,du =$$

6. OK, now let's practice using these formulas. Integrate each of the following. (Warning: maybe you don't need to use one of the new formulas.)

   (a) $\displaystyle\int \frac{1}{\sqrt{4x^2-9}}\,dx$

   (b) $\displaystyle\int \frac{x}{\sqrt{4x^2-9}}\,dx$

(c) $\displaystyle\int \frac{1}{x\sqrt{4x^2-9}}\,dx$

(d) $\displaystyle\int x\sqrt{4x^2+9}\,dx$

(e) $\displaystyle\int \frac{1}{9-4x^2}\,dx$

(f) $\displaystyle\int \frac{1}{4x^2+9}\,dx$