

Section 8.3: Trigonometric Substitution

Goal

In this section, we will introduce a technique of integration called *trigonometric substitution*. This technique is useful for dealing with functions containing the forms: $a^2 - u^2$, $a^2 + u^2$, and $u^2 - a^2$.

The 3 basic trig substitutions

form	substitution	identity	triangle
$a^2 - u^2$	$u = a \sin \theta$	$1 - \sin^2 \theta = \cos^2 \theta$	
$a^2 + u^2$	$u = a \tan \theta$	$1 + \tan^2 \theta = \sec^2 \theta$	
$u^2 - a^2$	$u = a \sec \theta$	$\sec^2 \theta - 1 = \tan^2 \theta$	

Important Note 1.

1. The pattern of constant², variable², and sign matches each identity.
2. You should always try u -sub before attempting trig sub. Also, sometimes you may already know a formula for evaluating the integral.

Examples

OK, let's jump in and do a bunch of examples.

Example 2. Integrate.

(a) $\int \frac{x^3}{\sqrt{1-x^2}} dx$

$$(b) \int \frac{1}{(4x^2 + 9)^2} dx$$

$$(c) \int \frac{\sqrt{x^2 - 25}}{x} dx$$

$$(d) \int \frac{1}{\sqrt{3 - 2x - x^2}} dx$$

$$(e) \int \frac{x}{\sqrt{x^2 - 25}} dx$$

$$(f) \int \frac{1}{\sqrt{x^2 - 25}} dx$$