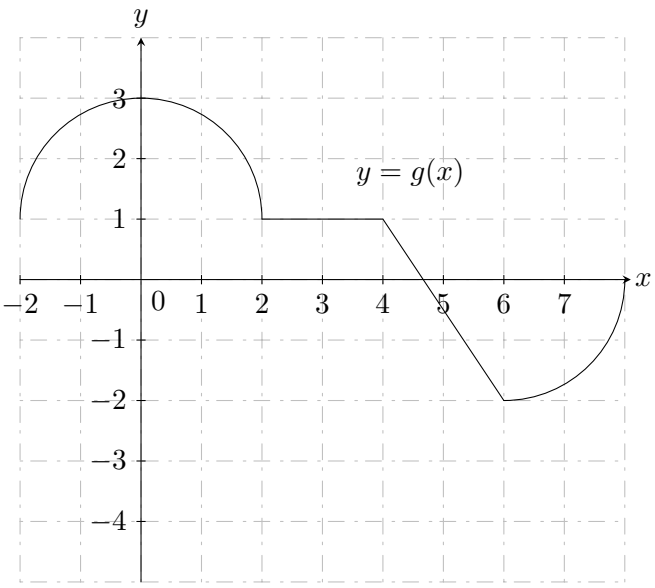
Intuitive Definite Integral

1.



(a) Find $\int_0^4 g(x)\ dx$.

(b) Find $\int_{-2}^8 g(x)\ dx$.

2. Use area of basic geometric shapes to find the following definite integrals.

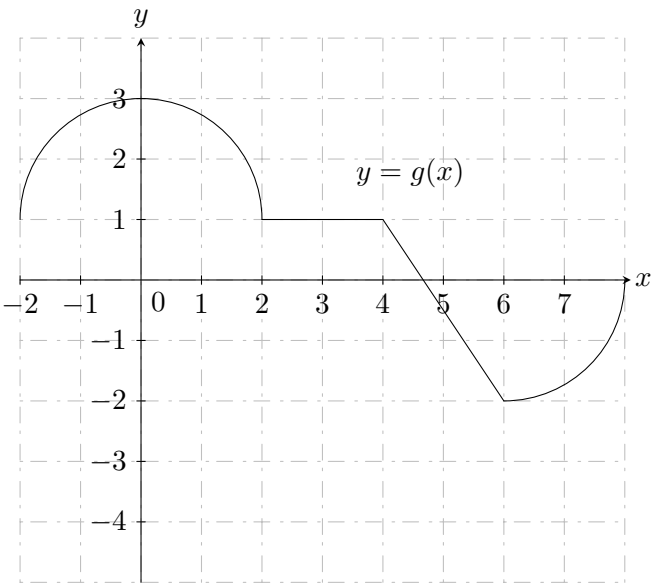
(a) Find $\int_{-2}^8 |x-3|\ dx$.

(b) Find $\int_{-3}^3 -\sqrt{9-x^2}\ dx$.

(c) Find $\int_0^2 \sqrt{1-\frac{x^2}{4}}\ dx$.

Riemann Sums

3. Estimate $\int_0^8 g(x)\,dx$ using 4 intervals and:



   (a) left end points.

   (b) right end points.

4. Find the left and the right sum of $f(x) = \sqrt{x} + 2$ on the interval [0,1] using 5 subdivisions.

5. The following sum: $3(\sqrt{5}+1) + 3(\sqrt{8}+1) + 3(\sqrt{11}+1) + 3(\sqrt{14}+1)$ is a right Riemann sum for a certain definite integral $\int_2^b f(x)\,dx$ using a partition of the interval $[2, b]$ into 4 subintervals of equal length.

   (a) What is b?

   (b) What is $f(x)$?

6. The following sum: $\frac{1}{1+\frac{2}{n}} \cdot \frac{2}{n} + \frac{1}{1+\frac{4}{n}} \cdot \frac{2}{n} + \frac{1}{1+\frac{6}{n}} \cdot \frac{2}{n} + \cdots + \frac{1}{1+\frac{2n}{n}} \cdot \frac{2}{n}$ is a right Riemann sum for a certain definite integral $\int_1^b f(x)\,dx$ using a partition of the interval $[1, b]$ into n subintervals of equal length.

   (a) What is b?

   (b) What is $f(x)$?

Fundamental Theorem of Calculus

7. Explain why the Fundamental Theorem of Calculus cannot be used to evaluate $\displaystyle\int_{-1}^{1} \frac{1}{x^2}\,dx$.

8. Compute each of the following definite integrals.

   (a) Let $A(x) = \displaystyle\int_0^x t^2 - t\,dt$. Find A'.

   (b) Let $f(x) = \displaystyle\int_0^x \sqrt[3]{t^2+1}\,dt$. Find f'.

   (c) Let $G(x) = \displaystyle\int_0^{x^2} t^3 \sin(t)\,dt$. Find G'.

   (d) Let $C(x) = \displaystyle\int_x^{x^3} \cos(\cos(t))\,dt$. Find C'.

9. Let $A(x) = \displaystyle\int_0^x \sin^2 t\,dt$. Determine where A attains a maximum value on the interval $[0, \pi]$.

Definite Integrals

10. Compute each of the following definite integrals.

    (a) $\displaystyle\int_0^1 x^2\,dx$

    (b) $\displaystyle\int_{-1}^1 x^4 - \frac{1}{2}x^3 + \frac{1}{4}x - 2\,dx$

    (c) $\displaystyle\int_0^\pi \sin(x)\,dx$

    (d) $\displaystyle\int_0^\pi \cos(2x)\,dx$

    (e) $\displaystyle\int_0^{\ln 2} e^{x/3}\,dx$

    (f) $\displaystyle\int_1^{e^2} \frac{x+1}{x^2}\,dx$

    (g) $\displaystyle\int_1^2 \frac{x^3 - 2\sqrt{x}}{x}\,dx$

    (h) $\displaystyle\int_0^{1/2} \frac{4}{\sqrt{1-x^2}}\,dx$

Indefinite Integrals

11. Compute each of the following indefinite integrals.

(a) $\int 5 \, dx$

(a) $\int 0 \, dx$

(b) $\int 2x^3 + x^2 - 5x + 5 \, dx$

(c) $\int -2\sqrt{x} \, dx$

(d) $\int \frac{x+1}{\sqrt{x}} \, dx$

(e) $\int \frac{1}{x^3} \, dx$

(f) $\int \frac{x+5}{x^2} \, dx$

(g) $\int \frac{\sin(x)}{\cos^2(x)} \, dx$

Substitution

12. Compute each of the following integrals.

(a) $\int (3x-1)^2 \, dx$ (Do 2 ways.)

(b) $\int (3x-1)^{99} \, dx$

(c) $\int 5x^2\sqrt{x^3-2} \, dx$

(d) $\int_0^2 xe^{x^2} \, dx$

(e) $\int \sin^2(x)\cos(x) \, dx$

(f) $\int_0^1 \frac{x}{x^2+1} \, dx$

(g) $\int x^2 \sec^2(x^3) \, dx$

(h) $\int \frac{x}{x^4+1} \, dx$

(i) $\int x\sqrt{x-1} \, dx$

Parts

13. Integrate each of the following.

    (a) $\int xe^{-x}\ dx$

    (b) $\int x^2 \sin(x)\ dx$

    (c) $\int \ln x\ dx$

    (d) $\int_0^1 \arctan(x)\ dx$

    (e) $\int x^3 e^{3x}\ dx$

    (f) $\int x^5 \sin(x^3)\ dx$

    (g) $\int e^x \cos(x)\ dx.$

Falling Objects

14. A skydiver steps out of an airplane. Her velocity in feet per second in the first 15 seconds of the fall can be represented by the function $f(x) = 30(1 - e^{-x/3})$. Find the distance fallen by the skydiver after 15 seconds have passed.

15. During the 2014 Flagstaff earthquake, a pinecone fell from a tree on the edge of a cliff, falling 215 meters.

    (a) How long did it take the piece of pinecone to hit the ground?

    (b) Ignoring air resistance, what will the velocity of the pinecone when it strikes the ground?

16. An person falls from the tallest building in Flagstaff and takes 3 seconds to reach the ground.

    (a) What is its speed at impact if air resistance is ignored?

    (b) How tall is the building?

    (c) What is the person's acceleration at the 2nd second?