MAT 136: Calculus I
Weekly Homework 2

NAME:

Instructions

You are allowed and encouraged to work together on homework. Yet, each student is expected to turn in his or her own work.

Reviewing material from previous courses and looking up definitions and theorems you may have forgotten is fair game. However, when it comes to completing assignments for this course, you should *not* look to resources outside the context of this course for help. That is, you should not be consulting the web, other texts, other faculty, or students outside of our course in an attempt to find solutions to the problems you are assigned. This includes Chegg and Course Hero. On the other hand, you may use each other, Discord, me, and your own intuition. **If you feel you need additional resources, please come talk to me and we will come up with an appropriate plan of action.** Please read NAU's Academic Integrity Policy.

Complete each of the following exercises. Your solutions should be complete and neatly written. In particular, you should show all of your work. Write your solutions on your own paper or prepare them digitally. This assignment is due on **Thursday, September 15** at class time.

Problems

1. Flash is running a 200-meter race. If he runs the first 175 meters at k meters per second and then runs the remaining 25 meters at ℓ meters per second, what would Flash's average speed be for the race?

2. When running a marathon you heard the timer call out 12 minutes as you passed the second mile-marker.

   (a) As you passed mile-marker 5 you heard the timer call out 33 minutes. What was your average speed from mile 2 to mile 5?

   (b) If you passed mile marker 5 at 33 minutes, what average speed do you need to run for the remainder of the race to meet your goal of completing the 26.2-mile marathon in 175 minutes? (Round your answer to two decimal places.)

3. Let f be given by the following graph. For (a)–(h), evaluate the given expression. For (i) and (j), find the indicated value. If a value does not exist, write DNE (for "does not exist"). You do *not* need to justify your answers.

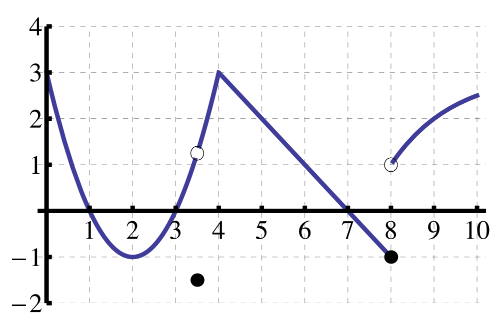

Figure 1: Graph of $y = f(x)$.

(a) $\lim_{x\to 3.5^-} f(x)$

(b) $\lim_{x\to 3.5^+} f(x)$

(c) $\lim_{x\to 3.5} f(x)$

(d) $f(3.5)$

(e) $\lim_{x\to 8^-} f(x)$

(f) $\lim_{x\to 8^+} f(x)$

(g) $\lim_{x\to 8} f(x)$

(h) $f(8)$

(i) Determine the slope of the tangent line at $x = 6$.

(j) Determine the slope of the tangent line at $x = 4$.

4. Let $f(x) = \dfrac{x^2 - 9}{x - 3}$ and $g(x) = x + 3$.

   (a) Is $f = g$? Explain your answer.

   (b) Is $\lim_{x\to 3} f(x) = \lim_{x\to 3} g(x)$? Explain your answer.

5. Consider the following function.

$$f(x) = \begin{cases} \dfrac{-1}{x-2}, & x > -1 \\ x^2 + 1, & x \leq -1 \end{cases}$$

   Evaluate each of the following expressions. If an expression does not exist, specify whether it equals ∞, $-\infty$, or simply does not exist (in which case, write DNE).

   (a) $f(-1)$

   (b) $\lim_{x\to -1^-} f(x)$

   (c) $\lim_{x\to -1^+} f(x)$

   (d) $\lim_{x\to -1} f(x)$

   (e) $f(2)$

   (f) $\lim_{x\to 2^+} f(x)$

6. Provide an example of a function for each of the following. You can either draw a graph or provide a formula for each example. If it is impossible to find an example, explain why.

   (a) $f(1)$ exists but $\lim_{x\to 1} f(x)$ does not exist.

   (b) $\lim_{x\to 1} f(x)$ exists but $f(1)$ does not exist.

   (c) Both $\lim_{x\to 1} f(x)$ and $f(1)$ exist but f is not continuous at $x = 1$.

   (d) f is continuous at $x = 1$ but one of $\lim_{x\to 1} f(x)$ or $f(1)$ does not exist.

7. Provide examples of functions f and g such that $\lim_{x\to 0}(f(x) + g(x))$ exists but neither $\lim_{x\to 0} f(x)$ nor $\lim_{x\to 0} g(x)$ exists.

8. Provide examples of functions f and g such that $\lim_{x\to 0}(f(x)g(x))$ exists but neither $\lim_{x\to 0} f(x)$ nor $\lim_{x\to 0} g(x)$ exists.

9. Suppose f is a function such that f has a tangent line at $x = a$. Provide an intuitive explanation (i.e., not a formal proof) as to why this implies that f is continuous at $x = a$.*

10. Find the values of the constants a and b that make the following function continuous.

$$f(x) = \begin{cases} \frac{1}{x}, & \text{for } x < -1 \\ ax + b, & \text{for } -1 \leq x \leq \frac{1}{2} \\ \frac{1}{x}, & \text{for } x > \frac{1}{2} \end{cases}$$

*On Weekly Homework 1, you provided an example of a function that was continuous at $x = 0$ but did not have a tangent line at $x = 0$. This shows that the converse of the stated claim is not true in general. That is, if a function f is continuous at $x = a$, the graph of f may or may not have a tangent line at $x = a$.