MAT 136: Calculus I Weekly Homework 3

NAME:

Instructions

You are allowed and encouraged to work together on homework. Yet, each student is expected to turn in his or her own work.

Reviewing material from previous courses and looking up definitions and theorems you may have forgotten is fair game. However, when it comes to completing assignments for this course, you should *not* look to resources outside the context of this course for help. That is, you should not be consulting the web, other texts, other faculty, or students outside of our course in an attempt to find solutions to the problems you are assigned. This includes Chegg and Course Hero. On the other hand, you may use each other, Discord, me, and your own intuition. If you feel you need additional resources, please come talk to me and we will come up with an appropriate plan of action. Please read NAU's Academic Integrity Policy.

Complete each of the following exercises. Your solutions should be complete and neatly written. In particular, you should show all of your work. Write your solutions on your own paper or prepare them digitally. This assignment is due on **Friday, September 30** at class time.

Problems

1. Use the Squeeze Theorem to evaluate the following limit. You should show sufficient justification.

$$\lim_{x \to 0^+} \sqrt{x} e^{\cos\left(\frac{\pi}{x}\right)}$$

Hint: First, notice that $-1 \le \cos\left(\frac{\pi}{x}\right) \le 1$. Next, find lower and upper bounds for $y = e^{\cos\left(\frac{\pi}{x}\right)}$.

- 2. Evaluate each of the following limits. If a limit does not exist, write DNE. Sufficient work must be shown and proper notation should be used. In particular, you should write limits where appropriate. Give *exact answers*.
 - (a) $\lim_{x \to \infty} \left(\sqrt{9x^3 + x} x^{3/2} \right)$ (b) $\lim_{x \to 0} \frac{\sec(x) - 1}{x}$ (c) $\lim_{x \to 0} \frac{1 - \cos(x)}{\sin(x)}$ (d) $\lim_{x \to \infty} (\ln(3x + 1) - \ln(x))$ (e) $\lim_{x \to \infty} \arctan(x)$

(f)
$$\lim_{x \to \infty} \arctan\left(\frac{1+x}{1-x}\right)$$

- 3. True or False? Circle the correct answer. You do not need to justify your answer.
 - (a) **True** or **False**: $\frac{d}{dx}[f(cx)] = c\frac{d}{dx}[f(x)].$
 - (b) **True** or **False**: $\frac{d}{dx} \left[f(x) \cdot g(x) \right] = f'(x) \cdot g'(x).$ (c) **True** or **False**: $\frac{d}{dx} \left[\frac{1}{f(x)} \right] = \frac{1}{f'(x)}.$
- 4. Consider the graph of the function g given in the figure below. Using the graph of g, sketch a possible graph of the derivative function g'.



5. The graph of a function g is given in the figure below. For each pair of values listed to the right of the graph, fill in the blank with exactly one of \langle , \rangle , or =.



6. Determine the derivative of each of the following functions using the limit definition of the derivative.

(a) $f(x) = 3x^2 - 4x + 17$ (b) $g(x) = \frac{1}{x+3}$ (c) $h(x) = \sqrt{x-2}$

- 7. Suppose the equation of the tangent line to the graph of some function f at the point x = 1 is given by y = 2x + 1.
 - (a) Find f'(1).
 - (b) Find f(1).
- 8. It is known that the derivative of $f(x) = x^3$ is $f'(x) = 3x^2$. Find an *equation* of the tangent line to the graph of f at 2. It does not matter what form the equation of your line takes.