MAT 136: Calculus I
Weekly Homework 5

NAME:

Instructions

You are allowed and encouraged to work together on homework. Yet, each student is expected to turn in his or her own work.

Reviewing material from previous courses and looking up definitions and theorems you may have forgotten is fair game. However, when it comes to completing assignments for this course, you should *not* look to resources outside the context of this course for help. That is, you should not be consulting the web, other texts, other faculty, or students outside of our course in an attempt to find solutions to the problems you are assigned. This includes Chegg and Course Hero. On the other hand, you may use each other, Discord, me, and your own intuition. **If you feel you need additional resources, please come talk to me and we will come up with an appropriate plan of action.** Please read NAU's Academic Integrity Policy.

Complete each of the following exercises. Your solutions should be complete and neatly written. In particular, you should show all of your work. Write your solutions on your own paper or prepare them digitally. This assignment is due on **Thursday, October 27** at class time.

Problems

1. Suppose f is a function such that its inverse function f^{-1} exists. Prove that $\frac{d}{dx}\left[f^{-1}(x)\right] = \frac{1}{f'(f^{-1}(x))}$.
   Hint: $f(f^{-1}(x)) = x$.

2. A water tank has the shape of an inverted circular cone (point down) with a base of radius 6 feet and a depth of 8 feet. Suppose that water is being pumped into the tank at a constant instantaneous rate of 4 cubic feet per minute. Find the instantaneous rate at which the water level is rising when the water in the tank is 3 feet deep. Give an *exact* answer, not a decimal approximation. Your answer should be labeled with appropriate units.

3. Use calculus to find the global maximum and global minimum of $f(x) = 2x + \frac{1}{2x}$ on the interval $[1, 4]$.

4. Let $f(x) = \frac{x}{x+2}$. It turns out that f satisfies the hypotheses of the Mean Value Theorem on the interval $[1, 4]$ (you do not need to show this). Find all the numbers c that the Mean Value Theorem guarantees exist.

5. Suppose f is continuous on $[4, 8]$ and $-2 \leq f'(x) \leq 4$ for all x in $(4, 8)$. Use the Mean Value Theorem to find lower and upper bounds for $f(8) - f(4)$. That is, find m and M such that $m \leq f(8) - f(4) \leq M$.

6. A truck driver handed in a ticket at a toll booth showing that in 2 hours they had covered 158 miles on a toll road with speed limit 70 mph. The driver was cited for speeding. Use the Mean Value Theorem to explain why. Be sure to state the assumptions that we have to make about the position function $p(t)$ of the truck to be able to apply the Mean Value Theorem.

 Written by D.C. Ernst