Quiz 5

Your Name:

Instructions

This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized*. Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 8 points for a total of 32 points. Good luck and have fun!

Part A

Complete **two** of the following problems.

A1. Show that in any group of 6 students there are 3 students who know each other or 3 students who do not know each other.

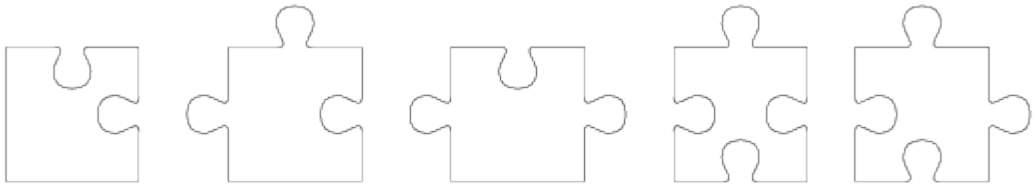
A2. In a PE class, everyone has 5 friends. Friendships are mutual. Two students in the class are appointed captains. The captains take turns selecting members for their teams, until everyone is selected. Prove that at the end of the selection process there are the same number of friendships within each team.

A3. Show that in any set of seven different positive integers there are three numbers such that the greatest common divisor of any two of them leaves the same remainder when divided by three.

Part B

Complete **two** of the following problems.

B1. You bought a rectangular puzzle consisting of 253 pieces. Each piece is identical to one of the 5 samples shown in the diagram. Is it possible to re-assemble this puzzle? If so, how many pieces of type E are there in the puzzle? If it's not possible, explain why. You may assume that the puzzle is solvable. *Hint:* 253 is divisible by 11.



B2. The inhabitants of a certain planet use not four but five basic arithmetic operations. The operations of addition, multiplication, subtraction and division are the same as ours, but they also have a special operation denoted by the sign @. We do not know exactly how this operation works, but we have found out that the following properties are valid for all x and y:

(a) $x@0 = x$

(b) $x@y = y@x$

(c) $(x+1)@y = (x@y) + y + 1$

What is the value of 12@5 on this planet?

B3. Below are the first four prototypes of a machine designed by elves designed to sort presents by weight (lightest on left, heaviest on right). Each machine sorts four presents at a time. The four presents are placed in the top, and then fall through the slides. Where two presents meet at a crossing, the lighter present goes to the left, and the heavier one goes to the right. This is repeated until all four presents are at the bottom.

Four elves, Fredi (39kg), Oswald (34kg), Iphis (28kg), and Esmeralda (21kg) are selected to simulate the presents in test runs. Which one of the four machines sorts the elves correctly for every possible order in which the elves can step into the four slides?