Quiz 6

Your Name:

Instructions

This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized*. Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 8 points for a total of 32 points. Good luck and have fun!

Part A

Complete **two** of the following problems.

A1. In the senate of the Klingon home world no senator has more than three enemies. Show that the senate can be separated into two houses so that nobody has more than one enemy in the same house.

A2. The front side of a paper square is white while its back side is black. The area of the square is 3 square inches. We folded one of the corners of the paper so that the corner is now on top of the diagonal of the square. Now the area of the black and white regions are the same. How far is the folded corner from the crease line?

A3. Prove that every natural number can be written as the sum of distinct powers of two.

Part B

Complete **two** of the following problems.

B1. 25 dogs and 25 cats sit around a circular table. Prove that it is always possible to find an animal both of whose neighbors are cats.

B2. Consider the following dialogue.

William: I have three children.

Harry: What are their ages?

William: The product of their ages is 36.

Harry: I still don't know their ages.

William: The sum of their ages is your apartment number.

Harry: I still don't know their ages!

William: The oldest plays football.

Harry: Now I know their ages.

What are the ages of William's children?

 Written by D.C. Ernst

B3. A frog jumps along the number line. It starts at 0 and every second it jumps n units to the right (the same positive integer n each time). We want to catch the frog. It's dark, we can't see the frog, and we do not know what n is. For all we know, it might be a super-frog, so n could be arbitrarily large. However, at any given second, we are allowed to choose an integer and search there. If the frog is on that integer, we catch it; if not, we have to try again. How can we catch the frog? We need to know which integer to check at each second. Justify your answer.