Quiz 2

Name:

Instructions

This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized*. Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 8 points for a total of 32 points. Good luck and have fun!

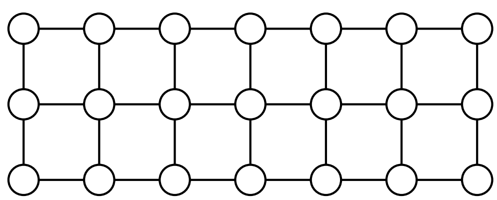
Part A

Complete **two** of the following problems.

A1. Show that in any group of 6 students there are 3 students who know each other or 3 students who do not know each other.

A2. My Uncle Robert owns a stable with 25 race horses. He wants to know which three are the fastest. He owns a race track that can accommodate five horses at a time. What is the minimum number of races required to determine the fastest three horses? *Note:* You do not need to justify that your solution attains a minimum.

A3. In the lattice below, we color 11 vertices points black. Prove that no matter which 11 are colored black, we always have a rectangle with black vertices (and vertical and horizontal sides).



Part B

Complete **two** of the following problems.

B1. Let a, b, c, d, e, f be distinct elements in the set $\{-3, -2, -1, 2, 4, 6\}$. What is the minimum possible value of $(a+b+c)^2 + (d+e+f)^2$?

B2. The Sylver Coinage Game is a game in which 2 players alternately name positive integers that are not the sum of nonnegative multiples of previously named integers. The person who names 1 is the loser! Here is a sample game between A and B:

(a) A opens with 5. Now neither player can name $5, 10, 15, \ldots$

(b) B names 4. Now neither player can name 4, 5, 8, 9, 10, or any number greater than 11.

(c) A names 11. Now the only remaining numbers are 1, 2, 3, 6, and 7.

(d) B names 6. Now the only remaining numbers are 1, 2, 3, and 7.

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(e) A names 7. Now the only remaining numbers are 1, 2, and 3.

(f) B names 2. Now the only remaining numbers are 1 and 3.

(g) A names 3, leaving only 1.

(h) B is forced to name 1 and loses.

If player A names 3, can you find a strategy that guarantees that the second player wins? If so, describe the strategy? If such a strategy is not possible, then explain why?

B3. The graph depicted below is an example of a Hastings Helm. Notice that we have labeled the 10 vertices of the graph with the natural numbers 1 through 10. We say that two vertices are *adjacent* if they are joined by an edge. For example, the vertex currently labeled by 4 is adjacent to the vertices labeled by 3, 5, and 10. Is it possible to shuffle the labels of the vertices so that the labels of adjacent vertices have no factors other than 1 in common? For example, using the current labeling, all the labels of the vertices adjacent to the vertex labeled by 2 have no nontrivial factors in common with 2. However, since the vertices labeled by 3 and 9 are adjacent and have a factor of 3 in common, the current labeling will not do the job. If you can find an appropriate labeling, then show it. If no such labeling exists, then explain why.