Quiz 3

Name:

Instructions

This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized.* Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 8 points for a total of 32 points. Good luck and have fun!

Part A

Complete **two** of the following problems.

A1. A mouse eats her way through a $3 \times 3 \times 3$ cube of cheese by tunneling through all of the 27 $1 \times 1 \times 1$ sub-cubes. If she starts at one corner and always moves to an uneaten sub cube, can she finish at the center of the cube?

A2. An overfull prison has decided to terminate some prisoners. The jailer comes up with a game for selecting who gets terminated. Here is his scheme. 10 prisoners are to be lined up all facing the same direction. On the back of each prisoner's head, the jailer places either a black or a red dot. Each prisoner can only see the color of the dot for all of the prisoners in front of them and the prisoners do not know how many of each color there are. The jailer may use all black dots, or perhaps he uses 3 red and 7 black, but the prisoners do not know. The jailer tells the prisoners that if a prisoner can guess the color of the dot on the back of their head, they will live, but if they guess incorrectly, they will be terminated. The jailer will call on them in order starting at the back of the line. Before lining up the prisoners and placing the dots, the jailer allows the prisoners 5 minutes to come up with a plan that will maximize their survival. What plan can the prisoners devise that will maximize the number of prisoners that survive? Some more info: each prisoner can hear the answer of the prisoner behind them and they will know whether the prisoner behind them has lived or died. Also, each prisoner can only respond with the word "black" or "red."

A3. In a PE class, everyone has 5 friends. Friendships are mutual. Two students in the class are appointed captains. The captains take turns selecting members for their teams, until everyone is selected. Prove that at the end of the selection process there are the same number of friendships within each team.

Part B

Complete **two** of the following problems.

B1. Suppose you have 5 coins, all identical in appearance and weight except for one that is either heavier or lighter than the other 4 coins. What is the minimum number of weighings one must do with a two-pan scale in order to identify the counterfeit and determine whether the counterfeit is heavier or lighter than the others? Describe your method in detail.

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B2. Consider our Star Base from Problem 37. Recall that our hyper drive allows us to jump from coordinates (a, b) to either coordinates $(a, a+b)$ or to coordinates $(a+b, b)$. If we start at $(1, 0)$, which points in the plane can we get to by using our hyper drive? Justify your answer.

B3. A soul swapping machine swaps the souls inside two bodies placed in the machine. Soon after the invention of the machine an unforeseen limitation is discovered: swapping only works on a pair of bodies once. Souls get more and more homesick as they spend time in another body and if a soul is not returned to its original body after a few days, it will kill its current host. Suppose the following sequence of soul swaps take place (where the names indicate the bodies that sit in the machine):

- Iron Man $\leftrightarrow$ Hulk
- Thor $\leftrightarrow$ Captain America

Find a way to put each soul back in the appropriate body using the minimum number of soul swaps possible. Your solution should be easy to interpret.

