Quiz 3

Your Name:

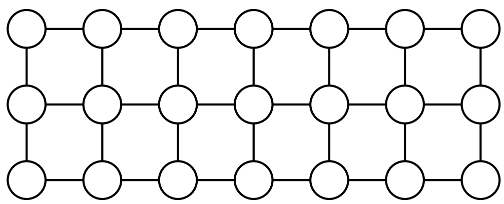
Instructions

This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized*. Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 4 points for a total of 16 points. Good luck and have fun!

Part A

Complete **two** of the following problems.

A1. In the lattice below, we color 11 vertices points black. Prove that no matter which 11 are colored black, we always have a rectangle with black vertices (and vertical and horizontal sides).



A2. Consider our Star Base from Problem 34. Recall that our hyper drive allows us to jump from coordinates (a, b) to either coordinates $(a, a+b)$ or to coordinates $(a+b, b)$. Assume we start at $(1, 1)$. Prove that if we can get to the point (a, b), then it must be the case that $\gcd(a, b) = 1$ (i.e., a and b only have a factor of 1 in common.

A3. Prove the following theorems in Circle-Dot. The axioms, rules of inference, as well as the theorems we proved in class are listed on the next page. You may only use an earlier theorem in the proof of a later theorem. That is, you may use Theorems C–H to prove Theorem I and you may use Theorems C–J to prove Theorem K if you desire.

Theorem I. ○ ○ ○○

Theorem K. ● ○ ○○

Part B

Complete **two** of the following problems.

B1. A certain store sells a product called widgets in boxes of $7, 9$, and 11. A number n is called *widgetable* if one can buy exactly n widgets by buying some number of boxes. What is the largest non-widgetable number?

B2. How many ways can 42 be written as the sum of 8 different positive integers?

B3. Determine whether we can obtain every sequence of length 3 in the Circle-Dot system. *Hint:* There are 8 sequences of length 3. You do not need to reproduce the proofs for sequences we've already obtained, but you should cite the appropriate theorem or proof.

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Circle-Dot

Circle-Dot begins with two words; called axioms. Using the two axioms and three rules of inference, we can create new Circle-Dot words, which are theorems in the Circle-Dot System. The process of creating Circle-Dot words using the axioms and rules of inference are proofs in the system. Below are the axioms for Circle-Dot. Note that $\circ$ and $\bullet$ are valid symbols in the system while w and v are variables that stand for any nonempty sequence of $\circ$'s and $\bullet$'s.

Axiom A. $\circ\bullet$

Axiom B. $\bullet\circ$

At any time in your proof, you may quote an axiom. Below are the rules for generating new statements from known statements.

Rule 1. Given wv and vw, conclude w

Rule 2. Given w and v, conclude $w \bullet v$

Rule 3. Given $wv\bullet$, conclude $w\circ$

In class, we proved the following theorems. You may only use an earlier theorem in the proof of a later theorem.

Theorem C. $\bullet$

Theorem D. $\circ$

Theorem E. $\bullet \bullet \bullet$

Theorem F. $\bullet \bullet \circ$

Theorem G. $\bullet \circ \circ$

Theorem H. $\circ \bullet \bullet\circ$

Theorem I. $\circ \circ \circ\circ$

Theorem J. $\bullet \circ \bullet$

Theorem K. $\bullet \circ \circ\circ$

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