Homework 11

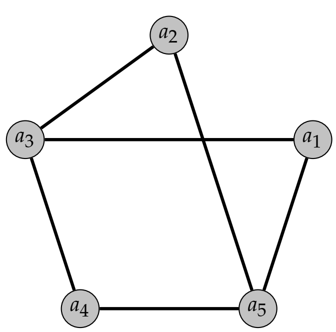
Discrete Mathematics

Please review the *Rules of the Game* from the syllabus. Reviewing material from previous courses and looking up definitions and theorems you may have forgotten is fair game. Since mathematical reasoning, problem solving, and critical thinking skills are part of the learning outcomes of this course, all assignments should be prepared by the student. Developing strong competencies in this area will prepare you to be a lifelong learner and give you an edge in a competitive workplace. When it comes to completing assignments for this course, unless explicitly told otherwise, you should *not* look to resources outside the context of this course for help. That is, you should *not* be consulting the web (e.g., Chegg and Course Hero), generative artificial intelligence tools (e.g., ChatGPT), mathematics assistive technologies (e.g., Wolfram Alpha and Photomath), other texts, other faculty, or students outside of our course in an attempt to find solutions to the problems you are assigned. On the other hand, you may use each other, the textbook, me, and your own intuition.You are highly encouraged to seek out assistance by asking questions in our Q&A Discussion Board in Canvas. You are allowed and encouraged to work together on homework. Yet, each student is expected to turn in their own work. **If you feel you need additional resources, please come talk to me and we will come up with an appropriate plan of action.**

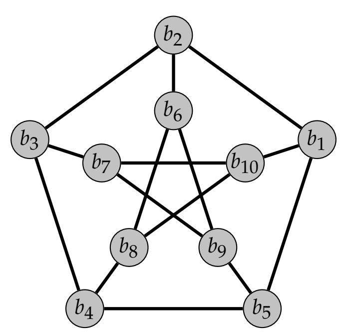
In general, late homework will not be accepted. However, you are allowed to turn in **up to three late homework assignments**. Unless you have made arrangements in advance with me, homework turned in after class will be considered late.

Complete the following problems. **Unless explicitly stated otherwise, you are expected to justify your answers.** In many problems this means that you should use words to describe what you are doing and why. In other problems, simply providing sufficient arithmetic may be sufficient. If a problem asks you to count something, please $\boxed{\text{box}}$ your final answer.

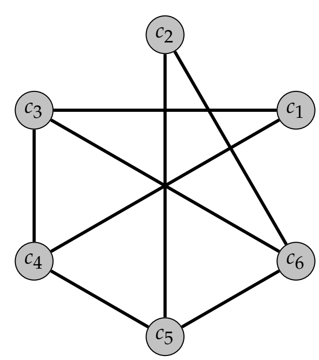
1. Consider a group of 5 people. Assume friendships are symmetric. That is, if Fred is friends with Maria, then Maria is friends with Fred.
   (a) Is it possible for everyone to be friends with exactly 2 of the people in the group? If so, provide an example. Otherwise, explain why this is impossible.
   (b) Is it possible for everyone to be friends with exactly 3 of the people in the group? If so, provide an example. Otherwise, explain why this is impossible.
2. Determine whether each graph below is bipartite. Explain your answer. If the graph is bipartite, list a bipartition.



(a)



(b)



(c)

3. How many edges does a graph have if it has the degree sequence 65555431?

4. Determine whether each of the following sequences is graphic. If the sequence is graphic, sketch a graph with this degree sequence. Otherwise give a convincing explanation that it is not possible.

   (a) 544321

   (b) 222211

   (c) 542211

   (d) 6333333

5. How many edges does each of the following have? Explain your answers.

   (a) $\overline{K_{4,3}}$

   (b) $\overline{C_n}$

6. Let $G = (V, E)$ be a simple graph of order n.

   (a) If $v \in V$, explain why $0 \leq \deg(v) \leq n-1$.

   (b) If $|V| \geq 2$, explain why we cannot have both a vertex of degree 0 and a vertex of degree $n-1$. Of course, we may not have either.

   (c) If $|V| \geq 2$, explain why there must be at least two vertices with the same degree. *Hint:* Use Parts (a) and (b). Consider three cases: (1) there is a vertex of degree 0, (2) there is vertex of degree $n-1$, (3) there is neither a vertex of degree 0 nor a vertex of degree $n-1$. In each case, apply the Pigeonhole Principle.

7. Suppose a shuffled deck of 52 regular playing cards are dealt into 13 piles of 4 cards each. Explain why it is possible to select one card from each pile to get one of each of the 13 card values Ace, 2, 3, . . . , 10, Jack, Queen, and King (as a set, not necessarily in that order). *Hint:* We can model the situation with a bipartite graph G with 13 vertices in the set V_1, each representing one of the 13 card values, and 13 vertices in the set V_2, each representing one of the 13 piles. There is an edge between a vertex $v_1 \in V_1$ and a vertex $v_2 \in V_2$ if a card with value v_1 is in pile v_2. Use Hall's Marriage Theorem.