Homework 11

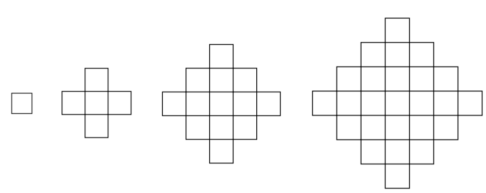
Discrete Mathematics

Please review the **Rules of the Game** from the syllabus. Reviewing material from previous courses and looking up definitions and theorems you may have forgotten is fair game. Since mathematical reasoning, problem solving, and critical thinking skills are part of the learning outcomes of this course, all assignments should be prepared by the student. Developing strong competencies in this area will prepare you to be a lifelong learner and give you an edge in a competitive workplace. When it comes to completing assignments for this course, unless explicitly told otherwise, you should *not* look to resources outside the context of this course for help. That is, you should *not* be consulting the web (e.g., Chegg and Course Hero), generative artificial intelligence tools (e.g., ChatGPT), mathematics assistive technologies (e.g., Wolfram Alpha and Photomath), other texts, other faculty, or students outside of our course in an attempt to find solutions to the problems you are assigned. On the other hand, you may use each other, the textbook, me, and your own intuition.You are highly encouraged to seek out assistance by asking questions on our Discord server. You are allowed and encouraged to work together on homework. Yet, each student is expected to turn in their own work. **If you feel you need additional resources, please come talk to me and we will come up with an appropriate plan of action.**

In general, late homework will not be accepted. However, you are allowed to turn in **up to two late homework assignments**. Unless you have made arrangements in advance with me, homework turned in after class will be considered late.

Complete the following problems. Unless explicitly stated otherwise, you are expected to justify your answers. In many problems this means that you should use words to describe what you are doing and why. In other problems, simply providing sufficient arithmetic may be sufficient. If a problem asks you to count something, please box your final answer.

1. **Show** that 4^n is a solution to the recurrence relation $a_n = 3a_{n-1} + 4a_{n-2}$.
2. **Solve** the following recurrence relations. **No detailed justification required.**
   (a) $a_n = a_{n-1} + 11$ with initial condition $a_1 = 7$.
   (b) $a_n = -5a_{n-1}$ with initial condition $a_1 = 40$.
3. Solve the recurrence relation $a_n = a_{n-1} + (2n - 1)$ with $a_1 = 1$. **No detailed justification required.**
4. Solve the recurrence relation $a_n = a_{n-1} + 2^n$ with $a_0 = 5$. **No detailed justification required.**
5. Consider the sequence of figures whose first few terms are as follows.



Let a_n be the area of the nth figure. Assume we start at $n = 1$, so that $a_1 = 1$, $a_2 = 5$, as so on.

(a) Find a recursive definition for the sequence. Don't forget any necessary initial conditions.

(b) Is the sequence arithmetic or geometric? If not, is it the sequence of partial sums of an arithmetic or geometric sequence?

(c) Find a closed formula for the nth term of the sequence.

6. Develop a recurrence relation with initial conditions for the number of ways to climb a staircase of n stairs by taking steps of one, two, or three stairs at a time. How many ways can a person climb a flight of 8 stairs under these constraints? **No detailed justification required.**

7. Let s_n denote the number of subsets of $[n]$ that contain no two consecutive elements. For example, $\{1,3,7\}$ is such a set, but $\{1,2,7\}$ is not since 1 and 2 are consecutive. **Determine** s_n in terms of a recurrence, a closed form, or by describing a bijection to a set we already know how to count.

8. (This problem is optional; do it if you want more practice) If you have enough toothpicks, you can make a large triangular grid. The size 1 grid requires 3 toothpicks, the size 2 grid requires 9 toothpicks, etc. Let t_n be the number of toothpicks required to make a size n triangular grid, so that $t_1 = 3$, $t_2 = 9$, and so on.

   (a) Find a recursive definition for the sequence. Don't forget any necessary initial conditions.

   (b) Is the sequence arithmetic or geometric? If not, is it the sequence of partial sums of an arithmetic or geometric sequence?

   (c) Find a closed formula for the nth term of the sequence.