

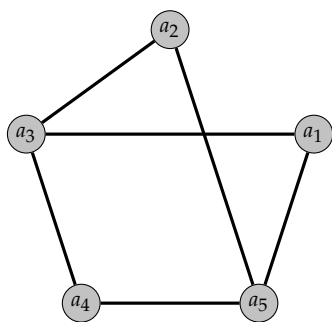
### Homework 13

#### Discrete Mathematics

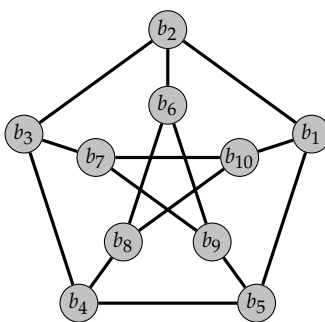
Please review the **Rules of the Game** from the syllabus.

Complete the following problems. Unless explicitly stated otherwise, you are expected to justify your answers. In many problems this means that you should use words to describe what you are doing and why. In other problems, simply providing sufficient arithmetic may be sufficient. If a problem asks you to count something, please box your final answer.

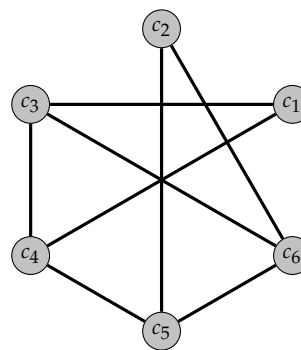
1. Consider a group of 5 people. Assume friendships are symmetric. That is, if Fred is friends with Maria, then Maria is friends with Fred.
  - (a) Is it possible for everyone to be friends with exactly 2 of the people in the group? If so, provide an example. Otherwise, explain why this is impossible.
  - (b) Is it possible for everyone to be friends with exactly 3 of the people in the group? If so, provide an example. Otherwise, explain why this is impossible.
2. Determine whether each graph below is bipartite. **Explain** your answer. If the graph is bipartite, list a bipartition.



(a)



(b)



(c)

3. How many edges does a graph have if it has the degree sequence 65555431? **No detailed justification required.**
4. Determine whether each of the following sequences is graphic. If the sequence is graphic, sketch a graph with this degree sequence. Otherwise give a convincing explanation that it is not possible.
  - (a) 544321
  - (b) 222211
  - (c) 542211
  - (d) 6333333
5. How many edges does each of the following have? **Explain** your answers.
  - (a)  $\overline{K_{4,3}}$
  - (b)  $\overline{C_n}$

6. Let  $G = (V, E)$  be a simple graph of order  $n$ .
- (a) If  $v \in V$ , **explain** why  $0 \leq \deg(v) \leq n - 1$ .
  - (b) If  $|V| \geq 2$ , **explain** why we cannot have both a vertex of degree 0 and a vertex of degree  $n - 1$ . Of course, we may not have either.
  - (c) If  $|V| \geq 2$ , **explain** why there must be at least two vertices with the same degree. *Hint:* Use Parts (a) and (b). Consider three cases: (1) there is a vertex of degree 0, (2) there is vertex of degree  $n - 1$ , (3) there is neither a vertex of degree 0 nor a vertex of degree  $n - 1$ . In each case, apply the Pigeonhole Principle.
7. Suppose a shuffled deck of 52 regular playing cards are dealt into 13 piles of 4 cards each. **Explain** why it is possible to select one card from each pile to get one of each of the 13 card values Ace, 2, 3,  $\dots$ , 10, Jack, Queen, and King (as a set, not necessarily in that order). *Hint:* We can model the situation with a bipartite graph  $G$  with 13 vertices in the set  $V_1$ , each representing one of the 13 card values, and 13 vertices in the set  $V_2$ , each representing one of the 13 piles. There is an edge between a vertex  $v_1 \in V_1$  and a vertex  $v_2 \in V_2$  if a card with value  $v_1$  is in pile  $v_2$ . Use Hall's Marriage Theorem.