Homework 5
Discrete Mathematics

Please review the **Rules of the Game** from the syllabus. Reviewing material from previous courses and looking up definitions and theorems you may have forgotten is fair game. Since mathematical reasoning, problem solving, and critical thinking skills are part of the learning outcomes of this course, all assignments should be prepared by the student. Developing strong competencies in this area will prepare you to be a lifelong learner and give you an edge in a competitive workplace. When it comes to completing assignments for this course, unless explicitly told otherwise, you should *not* look to resources outside the context of this course for help. That is, you should *not* be consulting the web (e.g., Chegg and Course Hero), generative artificial intelligence tools (e.g., ChatGPT), mathematics assistive technologies (e.g., Wolfram Alpha and Photomath), other texts, other faculty, or students outside of our course in an attempt to find solutions to the problems you are assigned. On the other hand, you may use each other, the textbook, me, and your own intuition.You are highly encouraged to seek out assistance by asking questions on our Discord server. You are allowed and encouraged to work together on homework. Yet, each student is expected to turn in their own work. **If you feel you need additional resources, please come talk to me and we will come up with an appropriate plan of action.**

In general, late homework will not be accepted. However, you are allowed to turn in **up to two late homework assignments**. Unless you have made arrangements in advance with me, homework turned in after class will be considered late.

Complete the following problems. Unless explicitly stated otherwise, you are expected to justify your answers. In many problems this means that you should use words to describe what you are doing and why. In other problems, simply providing sufficient arithmetic may be sufficient. If a problem asks you to count something, please $\boxed{\text{box}}$ your final answer.

1. Recall that a k-**permutation** of a set A is an injective function $w : [k] \to A$ and the set of all k-permutations of A is denoted by $S_{A,k}$. If A happens to be the set $[n]$, we use the notation $S_{n,k}$ and define $P(n,k) := |S_{n,k}|$. For $1 \le k \le n$, **establish a bijection** between the collection of nonattacking rook arrangements on an $n \times k$ chess board and the collection of k-permutations. A carefully drawn, but sufficiently general picture could be sufficient.

2. Recall the definition of Stirling numbers from Homework 2. **Explain** why $\begin{Bmatrix} n \\ n-1 \end{Bmatrix} = \dfrac{n(n-1)}{2}$.

3. Answer each of the following. **No detailed justification required**.

   (a) Draw function diagrams for every surjective (i.e., onto) function from $[4]$ to $[2]$.

   (b) How many surjective functions are there from $[n]$ to $[k]$ for $1 \le k \le n$? *Hint:* Your answer should involve Stirling numbers.

4. In how many ways can the letters of the word FLAGSTAFF be arranged? **No detailed justification required**.

5. A mathematics professor has 8 marker pens in her bag, and one day puts them along the bottom of the white board. How many ways can this be done if 3 are black, 2 are red, and there are 1 each of green, orange, and purple? **No detailed justification required**.

6. Imagine we have $n \geq 1$ people competing in a contest where ties are allowed.

   (a) **Explain** why the number of final rankings is given by

   $$1! \left\{ {n \atop 1} \right\} + 2! \left\{ {n \atop 2} \right\} + \cdots + n! \left\{ {n \atop n} \right\} = \sum_{k=1}^{n} k! \left\{ {n \atop k} \right\}.$$

   Note: You don't need to worry about justifying the expression on the righthand side. That is simply meant to be a reminder about how "Sigma notation" works.

   (b) Now, imagine that the people are numbered 1 through n. How many final rankings are possible under the proviso that person i's ranking is less than or equal to person $(i+1)$'s ranking? Such rankings are called **weakly increasing**. **Briefly explain your reasoning**.