Homework 9

Discrete Mathematics

Please review the **Rules of the Game** from the syllabus. Reviewing material from previous courses and looking up definitions and theorems you may have forgotten is fair game. Since mathematical reasoning, problem solving, and critical thinking skills are part of the learning outcomes of this course, all assignments should be prepared by the student. Developing strong competencies in this area will prepare you to be a lifelong learner and give you an edge in a competitive workplace. When it comes to completing assignments for this course, unless explicitly told otherwise, you should *not* look to resources outside the context of this course for help. That is, you should *not* be consulting the web (e.g., Chegg and Course Hero), generative artificial intelligence tools (e.g., ChatGPT), mathematics assistive technologies (e.g., Wolfram Alpha and Photomath), other texts, other faculty, or students outside of our course in an attempt to find solutions to the problems you are assigned. On the other hand, you may use each other, the textbook, me, and your own intuition.You are highly encouraged to seek out assistance by asking questions on our Discord server. You are allowed and encouraged to work together on homework. Yet, each student is expected to turn in their own work. **If you feel you need additional resources, please come talk to me and we will come up with an appropriate plan of action.**

In general, late homework will not be accepted. However, you are allowed to turn in **up to two late homework assignments**. Unless you have made arrangements in advance with me, homework turned in after class will be considered late.

Complete the following problems. Unless explicitly stated otherwise, you are expected to justify your answers. In many problems this means that you should use words to describe what you are doing and why. In other problems, simply providing sufficient arithmetic may be sufficient. If a problem asks you to count something, please $\boxed{\text{box}}$ your final answer.

1. How many nonnegative integer solutions are there for the equation $x_1+x_2+x_3+x_4+x_5+x_6 = 32$ such that $x_1 \leq 6$, $x_4 \leq 5$, and $x_6 \leq 8$? **For this problem, find the exact value of the answer and provide some justification.**

2. In any group of 6 people, **prove** that there is either a group of 3 who are known to each other or there are three who are total strangers.

   Hint: Pick one person, say person A, and think about the 5 others. Then by Pigeonhole Principle either at least 3 are known to A or at least 3 are strangers to A. Pick one of these scenarios (as the argument for the other is similar) and argue from there.

3. Consider the word PANDEMIC. **For this problem, find the exact value of the answer and provide some justification.**

   (a) How many arrangements of PANDEMIC are such that no letter is in its right place?

   (b) How many arrangements of PANDEMIC are such that exactly two letters are in their right place?

4. **Using induction, prove** that for all $n \in \mathbb{N}$, we have

$$\sum_{i=1}^{n} i(i+1) = \frac{n(n+1)(n+2)}{3}.$$

5. *Moved to next homework assignment.*

6. A **Dyck path** of length $2n$ is a lattice path from $(0,0)$ to (n,n) that takes n steps East from (i,j) to $(i+1,j)$ and n steps North from (i,j) to $(i,j+1)$ such that all points on the path satisfy $i \leq j$. This sounds more complicated that it really is. You can think of a Dyck path as one of our paths to get coffee that starts at $(0,0)$ and ends at (n,n) but never drops below the line $y = x$. Let $\text{Dyck}(n)$ denote set of all Dyck paths of length $2n$ and let $d_n := |\text{Dyck}(n)|$. We define $d_0 := 1$ for convenience. *Important:* Unfortunately, we also used d_n to denote the number of derangements of n. This problem is about Dyck paths, not derangements. Compute d_1, d_2, d_3, and d_4 via brute force by drawing the appropriate paths.