

Quiz 2

Your Name:

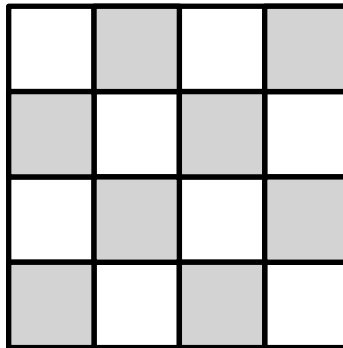
Instructions

This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized*. Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 4 points for a total of 16 points. Good luck and have fun!

Part A

Complete **two** of the following problems.

- A1. Imagine you have n^2 pebbles, each occupying one square on a $n \times n$ chess board. Suppose that each pebble must move to an adjacent square by only moving up, down, left, or right. For what values of n is the puzzle solvable? For what values of n is the puzzle unsolvable? Justify your answers by either providing a method for a solution or an explanation for why a solution is not possible.
- A2. Pennies and Paperclips is a two-player game played on a 4×4 checkerboard as shown below.



One player, “Penny”, gets two pennies as her pieces. The other player, “Clip”, gets a pile of paperclips as his pieces. Penny places her two pennies on any two different squares on the board. Once the pennies are placed, Clip attempts to cover the remainder of the board with paperclips - with each paperclip being required to cover two vertically or horizontally adjacent squares. Paperclips are not allowed to overlap. If the remainder of the board can be covered with paperclips then Clip is declared the winner. If the remainder of the board cannot be covered with paperclips then Penny is the winner.

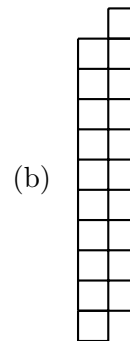
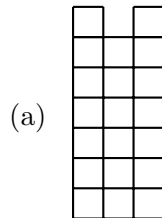
- (a) State and prove a conjecture that determines precisely every situation in which Penny wins based on the placement of the pennies.
- (b) State and prove a conjecture that determines precisely every situation in which Clip wins based on the placement of the pennies.
- A3. I have 10 sticks in my bag. The length of each stick is an integer. No matter which 3 sticks I try to use, I cannot make a triangle out of those sticks. What is the minimum length of the longest stick?

Part B

Complete **two** of the following problems.

B1. An overfull prison has decided to terminate some prisoners. The jailer comes up with a game for selecting who gets terminated. Here is his scheme. 10 prisoners are to be lined up all facing the same direction. On the back of each prisoner’s head, the jailer places either a black or a red dot. Each prisoner can only see the color of the dot for all of the prisoners in front of them and the prisoners do not know how many of each color there are. The jailer may use all black dots, or perhaps he uses 3 red and 7 black, but the prisoners do not know. The jailer tells the prisoners that if a prisoner can guess the color of the dot on the back of their head, they will live, but if they guess incorrectly, they will be terminated. The jailer will call on them in order starting at the back of the line. Before lining up the prisoners and placing the dots, the jailer allows the prisoners 5 minutes to come up with a plan that will maximize their survival. What plan can the prisoners devise that will maximize the number of prisoners that survive? Some more info: each prisoner can hear the answer of the prisoner behind them and they will know whether the prisoner behind them has lived or died. Also, each prisoner can only respond with the word “black” or “red.”

B2. Tile the following grids using every tetromino exactly once. If a tiling is not possible, explain why.



B3. How many ways can 42 be written as the sum of 8 different positive integers?