Quiz 5

Your Name:

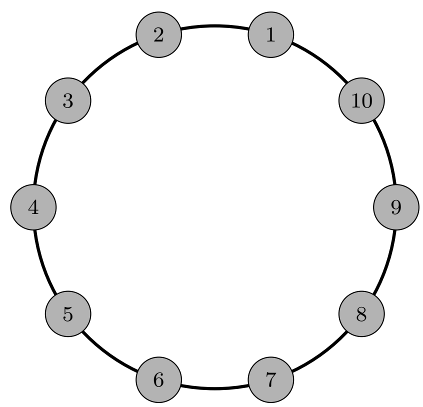
Instructions

This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized*. Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 4 points for a total of 16 points. Good luck and have fun!

Part A

Complete **two** of the following problems.

A1. Ten people form a circle. Each picks a number and tells it to the two neighbors adjacent to him/her in the circle. Then each person computes and announces the average of the numbers of his/her two neighbors. The figure shows the average announced by each person. What is the number picked by the person who announced 6?



A2. Let X be the intersection of the diagonals of the trapezoid $ABCD$ with parallel sides AB and CD. Show that the areas of triangles AXD and BXC are the same.

A3. There are 30 red, 40 yellow, 50 blue, and 60 green balls in a box. We take out balls from the box with closed eyes. On the first turn we take out 1 ball, on the second turn we take out 2, and so on. On the nth turn we take out n balls. What is the minimum number of balls we need to take out to guarantee the following:

(a) We have a blue ball;

(b) We have all four colors.

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Part B

Complete **two** of the following problems.

B1. A frog jumps along the number line. It starts at 0 and every second it jumps n units (the same positive integer n each time). In addition, the frog is allowed to start by going either to the left or to the right; once it chooses a direction, it always jumps n units in that direction. We want to catch the frog. It's dark, we can't see the frog, and we do not know what n is. For all we know, it might be a super-frog, so n could be arbitrarily large. However, at any given second, we are allowed to choose an integer and search there. If the frog is on that integer, we catch it; if not, we have to try again. Describe a method for catching the frog.

B2. A town of Smurfs consists of 24 blue, 8 pink, and 16 purple individuals. When two Smurts of different colors shake hands, they both change their colors to the third color. Is it possible that all Smurfs in the town eventually have the same color? If so, describe a pattern of handshakes that will convert all the Smurfs to the same color. If this is not possible, provide a detailed argument as to why.

B3. Suppose you have 9 coins, all identical in appearance and weight except for one that **we know is heavier** than the other 8 coins. Is it possible to detect the counterfeit coin in at most two weighings with a two-pan scale? If so, describe an algorithm and explain why it works. If this is impossible, explain why.

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